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## Precautionary Demand for Money in a Monetary Business Cycle Model<sup>\*</sup>

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### Abstract

We investigate quantitative implications of precautionary demand for money for business cycle dynamics of velocity and other nominal aggregates. Accounting for such dynamics is a standing challenge in monetary macroeconomics: standard business cycle models that have incorporated money have failed to generate realistic predictions in this regard. In those models, the only uncertainty affecting money demand is aggregate. We investigate a model with uninsurable *idiosyncratic* uncertainty about liquidity need and find that the resulting precautionary motive for holding money produces substantial qualitative and quantitative improvements in accounting for business cycle behavior of nominal variables, at no cost to real variables.

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# 1 Introduction

In this paper, we study, theoretically and quantitatively, aggregate business cycle implications of precautionary demand for money. It is an outstanding challenge in the literature to account for business cycle behavior of nominal aggregates and their interaction with real aggregates. Business cycle models that have tried to incorporate money explicitly through, for example, cash-in-advance constraints, have done so while assuming that agents face only aggregate risk, which has resulted in the demand for money being largely deterministic, in the sense that the cash-in-advance constraint almost always binds. Such models have unrealistic implications for the dynamics of nominal variables, as well as for interaction between real and nominal variables, when compared to data (see, e.g., Cooley and Hansen, 1995).

Yet precautionary motive for holding liquidity seems to be strong in the data, and its nature suggests that *idiosyncratic* risk may play a key role for money demand, as shown in Telyukova (2011). In that paper, it is documented that the median household has about 50% more liquidity than it spends on average per month, and that controlling for observables, consumption of goods requiring a liquid payment method (e.g. cash or check) exhibits volatility consistent with the presence of significant idiosyncratic risk. Thus, aggregate implications of idiosyncratic risk and resulting precautionary money demand are important to investigate, especially given the unresolved questions regarding monetary business cycles. The goal of this paper is to conduct such an investigation. The set of questions we want to answer is: What are the aggregate implications of precautionary demand for money? Can it help account for business cycle dynamics of velocity of money, interest rates and inflation, and their interaction with real variables?

Existing monetary business cycle models that incorporate money demand via a cash-in-advance constraint, such as cash-credit good models, calibrated to aggregate data, cannot account for aggregate facts such as variability of velocity of money, correlation of velocity with output growth or money growth, correlation of inflation with nominal interest rates, and others, as Hodrick, Kocherlakota and Lucas (1991) have shown. The reason is that in such models, the only type of uncertainty the households face is aggregate uncertainty. The magnitude of this uncertainty in the data is not large enough to generate significant precautionary motive for holding money in the model, so that the cash-in-advance constraint almost always binds. Then, money demand in the model is made equivalent to cash-good consumption, tightly linking volatility of money demand to volatility of aggregate consumption. Aggregate consumption, in turn, is not volatile enough in the data to generate observed volatility of money demand (or its inverse, velocity) or other nominal aggregates.

We show that incorporating precautionary demand for money generated by unpredictable idiosyncratic variation, in combination with aggregate uncertainty, makes a crucial difference in the ability of the model to account for monetary facts mentioned above, by breaking the link between money demand and aggregate consumption. Agents generally hold more money than they spend, so that money demand is now linked only to consumption of agents whose preference shock realizations make them spend all of their money in trade. We show that velocity of money can be significantly more volatile in this heterogeneous-agent setting, thanks to the *unconstrained* agents, who are absent in previous models with only aggregate risk. The presence of both constrained and unconstrained households is key to the qualitative and quantitative results. In other words, idiosyncratic risk in this context does not average out in a way that can be adequately captured by a representative agent model, as Hodrick et al (1991) in fact anticipated in the discussion of their results (p. 380). In addition, the magnitude of idiosyncratic volatility is much higher than aggregate volatility: the standard deviation of aggregate consumption is 0.5%; we will measure the standard deviation of idiosyncratic consumption shocks to be around 19%.

Introducing idiosyncratic risk into the model also changes the nature of the inflation tax, and thus has an impact on welfare costs of inflation. In the standard cash-credit good model with only aggregate risk, where the cash-in-advance constraint in practice always binds, the cost to agents of having a positive nominal interest rate is in having to hold money to spend on the cash good. In standard cash-credit good models as in our model, the nominal interest rate drives a wedge between the marginal rate of transformation and the marginal rate of substitution between cash and credit goods.<sup>1</sup> Without idiosyncratic shocks, this wedge affects all households equally: taken as given the total of cash goods transacted, the allocation among households equates the marginal utilities among households. Once we add uninsurable idiosyncratic risk, not only is the total quantity of goods in the cash market suboptimal with inflation, but the allocation of this quantity among households is also inefficient: it does not equate the household's marginal utilities in the cash market. The agents whose idiosyncratic shock realization is binding have a marginal utility is higher than that of the unconstrained agents, and thus, ex post, bear more of the cost of inflation. We show in our setting that this raises the cost of inflation. The nature of the inflation tax in the model with idiosyncratic risk also depends on whether inflation increases are anticipated or not. In the latter case, as we show in our welfare analysis, a surprise increase in inflation can further exacerbate this last distortion.

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<sup>1</sup>In our model, the timing is such that it is the firms who take cash from one period to the next, after selling the cash goods. Whether discounting occurs before or after the cash market, however, is immaterial for the wedge between marginal rate of transformation and marginal rate of substitution. Given free entry of firms, the full incidence of the inflation tax is still born by the households.

We study these mechanisms qualitatively and quantitatively in a model that combines, in each period, two types of markets in a sequential manner, and where both aggregate and idiosyncratic uncertainty are present. The first-subperiod market is a standard Walrasian market, which we will term, somewhat loosely, the “credit market”. The second market is also competitive, but characterized by anonymity and the absence of barter possibility, which makes a medium of exchange - money - essential in trade. We term this market the “cash market”.<sup>2</sup> The credit market is much like a standard real business cycle model, with the production function being subject to aggregate productivity shocks. Two features distinguish this market from the RBC framework. First, households have to decide how much money to carry out of this market for future cash consumption. Second, part of the output in the credit market is carried into the cash market by retail firms, who buy these goods on credit and subsequently transform them into cash goods. This introduces an explicit link between the real and monetary sectors of the economy, as credit-market capital becomes indirectly productive in the cash market.

At the start of the second-subperiod cash market, agents are subject to uninsurable idiosyncratic preference shocks which determine how much of the cash good they want to consume, but the realization of the shock is not known at the time that agents make their portfolio decisions. This generates precautionary motive for holding liquidity. In our model, we show analytically how the idiosyncratic shocks, and the resulting heterogeneity of households, result in amplified dynamics of velocity of money. We also show that absent idiosyncratic shocks, the model produces counterfactual nominal dynamics for values of the coefficient of relative risk aversion in the standard range in RBC literature.

Another contribution of our work is the calibration of the model. To our knowledge, all the existing models of the types mentioned above that have looked at aggregate behavior of nominal variables have been calibrated to aggregate data. Instead, we also use micro survey data on liquid consumption from the Consumption Expenditure Survey, like in Telyukova (2011), to calibrate idiosyncratic preference risk in our cash market. In general, preference risk of the type that creates precautionary liquidity demand has not been measured in calibration of other aggregate models, and in the few contexts where precautionary liquidity demand has appeared, it has been treated as a free parameter (e.g. Faig and Jerez, 2007). Our use of micro data tightly disciplines our calibration.

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<sup>2</sup>This setup is consistent with both cash-credit good models a la Lucas and Stokey (1987) and monetary search models in the style of Lagos and Wright (2005). In theory, money-search-style idiosyncratic matching shocks could be interpreted as a type of idiosyncratic preference shock (Wallace 2001). However, with matching shocks, agents spend all or none of their money, while a crucial part of our argument is that a preference shock may cause a household to spend only part of their money holdings. The natural empirical counterparts of the two types of shocks are also different.

Once calibrated, we compute the model to investigate the effects of real productivity shocks and monetary policy shocks. We find that precautionary demand for money makes a dramatic difference in helping the model account for a variety of dynamic moments of nominal aggregates in the data. We test these results by also computing a version of the model where we shut down the idiosyncratic risk, and find that without it, the model is incapable of reproducing key nominal moments in the data, much as previous literature has suggested.

Our results lead us to conclude that in many monetary contexts, especially those aimed at accounting for aggregate data facts, it is important not to omit idiosyncratic uncertainty that gives rise to precautionary demand for money. As one example, omitting this empirically relevant mechanism may cause the standard practice of calibrating monetary models to the aggregate money demand equation, as has been done in many cash-in-advance models and monetary search models, to produce misleading results for parameters and counterfactual quantitative implications. We demonstrate this by calibrating a version of the model *without* idiosyncratic shocks to target some data properties of aggregate money demand.

This paper is related to several strands of literature. On the topic of precautionary demand for liquidity,<sup>3</sup> the key mechanism in our model is close to Faig and Jerez (2007), Telyukova and Wright (2008) and Telyukova (2011). In Telyukova and Wright (2008) and Telyukova (2011), the idiosyncratic uncertainty about liquidity need is shown, respectively theoretically and quantitatively, to be relevant for household portfolio decisions to hold liquid assets and credit card debt simultaneously. Faig and Jerez (2007) look at the behavior of velocity and nominal interest rates over the long run. They find that with precautionary liquidity demand, the simulated time series of velocity over the last century, interpreted as a series of steady states, fits the empirical series well. Lagos and Rocheteau (2005) study steady state properties of a monetary economy with idiosyncratic preference shocks.<sup>4</sup> On the broad subject of accounting for aggregate behavior of nominal variables, a recent paper is Wang and Shi (2006). In their model, however, search intensity is the key mechanism behind velocity fluctuations over the business cycle.

The paper is organized as follows. Section 2 describes the model and characterizes the equilibrium. Section 3 demonstrates analytically the impact of precautionary demand for money on

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<sup>3</sup>The subject of precautionary money demand goes back to at least Keynes(1936), who defined its reason as “to provide for contingencies requiring sudden expenditure and for unforeseen opportunities of advantageous purchases”. Precautionary demand for money is often modeled in Baumol-Tobin-style inventory-theoretic models, from Whalen (1966) to fully dynamic stochastic models such as Alvarez and Lippi (2009). Uninsurable idiosyncratic liquidity shocks are also an essential element of models based on Diamond and Dybvig (1983). Lucas (1980) studies the equilibrium in a cash-in-advance model with precautionary demand for money.

<sup>4</sup>In another paper on the broad subject of precautionary money demand, Hagedorn (2008) shows that strong liquidity effects can arise when precautionary demand for money is taken into account in a cash-credit good model with banking.

the dynamic behavior of money, velocity and interest rates, and discusses the inflation tax further. Section 4 describes our calibration strategy for several versions of the model. In section 5 we present our results and discuss the quantitative role of precautionary liquidity demand; we then discuss how omission of precautionary demand may lead model calibration and implications astray. In section 6 we show how precautionary demand affects welfare costs of inflation, both anticipated and unanticipated. Section 7 concludes.

## 2 Model

The economy is populated by a measure 1 of infinitely-lived households, who rent labor and capital to firms, consume goods bought from the firms, and save. There are two types of markets open sequentially during the period. In the first subperiod, a Walrasian market is open, in which all parties involved in transactions are known and all trades can be enforced. In the second subperiod, the market is competitive, in the sense that all agents are price takers, but we assume that money is essential in trade.<sup>5</sup> Since in the first-subperiod market households pay with either cash or credit for consumption, and as we discuss below, retail firms buy on credit, we will refer to this as the “credit market”,<sup>6</sup> while the second subperiod - where payment takes place using money only - will be termed the “cash market”.

There are two types of firms in the economy. Production firms use capital and labor as inputs in production, and their output is used for consumption and capital investment in the credit market. However, part of the output is also bought in the credit market by retail firms, who then transform the goods one-for-one into retail goods to be sold in the cash market.<sup>7</sup>

### 2.1 Households

Households maximize lifetime expected discounted utility,

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( U(c_t) - Ah_t + \vartheta_t u(q_{\vartheta,t}) \right) \right] \quad (1)$$

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<sup>5</sup>Temzelides and Yu (2004) derive sufficient conditions under which money is essential in competitive markets. See also Levine (1991) and Rocheteau and Wright (2005).

<sup>6</sup>In the data, credit cards are a complex arrangement that for some households is a convenience device, while for others, a long-term revolving credit arrangement. Our credit market is a simplification, and not meant to capture credit cards tightly; however, we believe it to be the most natural simplification, and we will use survey data to discipline cash consumption as goods which cannot be paid by credit.

<sup>7</sup>Our retailers are not meant to correspond one-for-one to the retail sector in the data: some retailers in the data are better characterized as selling in the credit market, whereas the cash sector includes firms that are not retailers.

where  $0 < \beta < 1$ . While here for analytical tractability we assume that the utility function is separable, in computation we examine cases with both separable and CES utility functions.<sup>8</sup> Inada conditions on  $U(\cdot)$  and  $u(\cdot)$  ensure that consumers participate in both markets. Utility achieved in each period, depends on consumption  $c_t$  and time spent working  $h_t \in [0, 1]$  in the first subperiod, and in the second subperiod, consumption  $q_t$  and the preference shock  $\vartheta_t$ . First-subperiod utility follows the Hansen-Rogerson specification of indivisible labor with lotteries. The taste shock  $\vartheta_t$  is realized when the credit market is already closed and money holdings can no longer be adjusted, as described below. This will lead to precautionary demand for money.

We normalize household money holdings and nominal bond holdings by the aggregate money holdings, rendering them stationary. Households start the period with normalized money holdings  $m_t$  and choose  $\tilde{m}_t$  normalized money to bring into the cash market, before  $\vartheta_t$  occurs. Households also own capital  $k_t$  and normalized nominal bonds  $b_t$  sold to them by retail firms, as detailed below.<sup>9</sup> Let wage, capital rent, and the return on nominal bonds be  $w_t, r_t, i_{t-1}$ . Let the nominal price of the credit good be  $p_t$ . The budget constraint, expressed in nominal terms, is

$$m_t + (1 + r_t - \delta)p_t k_t + w_t p_t h_t + b_t(1 + i_{t-1}) = p_t c_t + \tilde{m}_t + p_t k_{t+1} + b_{t+1} \quad (2)$$

We will later employ  $\phi_t = 1/p_t$  to denote the value of one unit of money in terms of the credit good, in order to rewrite the budget constraint in real terms. Given nominal price  $\psi_t$  of the cash good, consumption  $q_{\vartheta,t}$  in the cash market, conditional on the preference shock realization  $\vartheta_t$ , has to satisfy  $\psi_t q_{\vartheta,t} \leq \tilde{m}_t$ .

## 2.2 Firms

The problem of the *production* firm is standard – to maximize its profits in each period. Given a constant returns to scale production function  $y_t = e^{z_t} f(k_t, h_t)$ , where  $z_t$  is stochastic productivity level, the problem is:  $\max_{k_t, h_t} \{e^{z_t} f(k_t, h_t) - w_t h_t - r_t k_t\}$ . The solution is characterized by the usual first-order conditions.

*Retail* firms exist for two periods: they buy the goods in the credit market, selling nominal bonds to households to do so, sell the goods in the subsequent cash market, and settle their debt in the following credit market, before disbanding. Free entry in the retail market yields the following

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<sup>8</sup>Given that the leisure component of utility is linear, results along the same lines as presented go through for the CES case, at some notational cost.

<sup>9</sup>In principle, households can hold shares of firms as well. We will see that in our formulation all firms make zero profits, share holding is irrelevant. Alternatively, we can formulate the economy with firms selling shares instead of bonds; this leads to equivalent allocations of resources, but involves more notation.

condition, expressed in nominal terms at time  $t$ :

$$\Pi_{\pi t} = \max_{q_t} \left( \frac{\psi_t q_t}{1 + i_t} - \frac{q_t}{\phi_t} \right) = 0. \quad (3)$$

All cash receipts from retail sales go towards repayment in the following credit market; the value of this repayment is discounted using the nominal interest rate. The repayment equals the current nominal value for the  $q_t$  goods that were purchased in the credit market by the retailers. Since the cash market is competitive, retail firms sell all their goods in equilibrium.<sup>10</sup>

### 2.3 Monetary Policy and Aggregate Shocks

The monetary authority follows an interest rate feedback rule

$$\frac{1 + i_{t+1}}{1 + \bar{i}} = \left( \frac{1 + i_t}{1 + \bar{i}} \right)^{\xi_{ii}} \left( \frac{1 + \pi_t}{1 + \bar{\pi}} \right)^{\xi_{i\pi}} \left( \frac{y_t}{\bar{y}} \right)^{\xi_{iy}} \exp(\varepsilon_{t+1}^{mp}). \quad (4)$$

The variables with bars denote central bank's long-run target levels of output, inflation and the nominal interest rate. The term  $\varepsilon_t^{mp}$  denotes a stochastic monetary policy shock which is realized at the beginning of period  $t$ . At the end of the period, the government makes a lump sum transfer  $\varpi_t M_t$ , where  $M_t$  is the aggregate money stock. The rate of money supply growth  $\varpi_t$  is adjusted by the central bank to make  $i_t$  arise as the equilibrium price.

The second, independent, aggregate shock process is on the productivity level  $z_t$ . As is standard in business cycle literature,  $z_t$  follows an AR(1) of the form  $z_{t+1} = \xi_{zz} z_t + \varepsilon_{t+1}^z$ .

### 2.4 Recursive Formulation of the Household Problem

From now on, we will conserve notation by omitting time subscripts, and using primes to denote  $t + 1$ . The aggregate state variables in this economy are  $S = (K, z, i_{-1}, i, (1 + \varpi_{-1})\phi_{-1})$ : the aggregate capital stock, the technology shock, previous interest rate in the economy, current interest rate, and the previous period's post-injection real value of money, which households need to determine the current rate of inflation. The individual state variables at the beginning of the credit market are  $(m, k, b)$ : normalized money holdings, capital and bond holdings. We take the credit good as the numeraire. The household solves

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<sup>10</sup>We assume that goods are storable, so even at zero expected real interest rate, this is without loss of generality.



$$V(k, m, b, S) = \max_{c, h, \tilde{m}, k', b', \{q_\vartheta\}} \left\{ U(c) - Ah \right. \quad (5)$$

$$\left. + \mathbb{E}_\vartheta \left[ \vartheta u(q_\vartheta) + \beta \mathbb{E}_{z', i'} V(k', m', \frac{b'}{1 + \varpi}, S') \right] \right\}$$

$$\text{s.t. } c + \phi \tilde{m} + k' + \phi b' = \phi m + \phi b(1 + i_{-1}) + (1 + r - \delta)k + wh \quad (6)$$

$$\psi q_\vartheta \leq \tilde{m} \quad (7)$$

$$\pi = \frac{(1 + \varpi_{-1})\phi_{-1}}{\phi} \quad (8)$$

$$1 + \varpi = \Omega(S) \quad (9)$$

$$m' = \frac{\tilde{m}}{1 + \varpi} - \frac{\psi q_\vartheta}{1 + \varpi} + \frac{\varpi}{1 + \varpi} \quad (10)$$

$$z' = \xi_{zz}z + \varepsilon'_1 \quad (11)$$

$$\widetilde{(1 + i')} = \xi_{ii}(\widetilde{1 + i}) + \xi_{i\pi}\tilde{\pi} + \xi_{iy}\tilde{y} + \varepsilon'_2 \quad (12)$$

The interest rate rule here is given in short hand, with  $\tilde{x}$  referring to log-deviations of the variable  $x$  from its target level. Denote the household state variables as  $s = (k, m, b)$ . Denote the policy functions of the household's problem by  $g(s, S)$ , with  $g_x(\cdot)$  as the policy function for the choice variable  $x$ .

**Definition 1.** A Symmetric Stationary Monetary Equilibrium is a set of pricing functions  $\phi(S)$ ,  $\psi(S)$ ,  $w(S)$ ,  $r(S)$ ; law of motion  $K'(S)$ , value function  $V(s, S)$  and policy functions  $g_c(s, S)$ ,  $g_h(s, S)$ ,  $g_k(s, S)$ ,  $g_b(s, S)$ ,  $g_m(s, S)$ ,  $\{g_{q, \vartheta}(s, S)\}$ , all  $\vartheta$ , such that: (i) households optimize by solving (5), given prices and laws of motion; (ii) production and retail firms optimize, as in section 2.2; (iii) free entry of retailers implies  $\Pi_r = 0$ ; (iv) the aggregate law of motion follows from the sum of all individual decisions:  $K'(S) = \int_0^1 g_k^i(s, S) di$ . Finally, (v) all markets clear:

$$\begin{aligned} \int_0^1 g_m^i(s, S) di &= 1 \\ \int_0^1 \phi(S) g_b^i(s, S) di &= \int_0^1 [\mathbb{E}_\vartheta g_{q, \vartheta}^i(s, S)] di \\ \int_0^1 g_h^i(s, S) di &= H(S) \\ (1 - \delta)K + e^z f(H(S), K) &= \int_0^1 g_c^i(s, S) di + K'(S) + \int_0^1 [\mathbb{E}_\vartheta g_{q, \vartheta}^i(s, S)] di \end{aligned}$$

In appendix A we derive and analyze the equilibrium conditions of the problem, to show that the quasi-linear specification of the utility function allows equilibria in which the distribution of wealth created by the idiosyncratic shocks in the cash market washes out in the following credit market, as long as  $h$  remains in the interior. The following result is immediate:

**Result 1.** The choice of  $c, \tilde{m}, k', b'$  only depends on the aggregate states  $S$ .

Further, we show that for low enough realizations of the shock  $\vartheta$ , cash balances are not spent in full, and that the resulting  $q_\vartheta$  is the efficient quantity that equalizes the marginal utilities of credit- and cash-good consumption. In contrast, if a shock  $\hat{\vartheta}$  results in a binding cash constraint, then for any  $\vartheta > \hat{\vartheta}$ , the constraint will also bind, which will lead to the underconsumption of the cash good relative to social optimum.

What does this say about consumer payment behavior? In the data, it is reasonable to expect that different consumers make different choices with respect to how much to consume with cash versus credit. In our model, the cash market is meant to capture transactions which cannot be paid using credit, as we will discuss in the calibration section. However, the “credit” market in the first subperiod is more flexible: the model is silent on whether households pay using credit or cash, as these methods can be costlessly transferred one into the other. Thus the model implicitly can accommodate the kind of heterogeneity in portfolios and payment methods seen in the data. In order to discipline our model, we will, in calibration, match the total volume of consumer transactions done by liquid payment methods (cash, check, debit) in aggregate data. Since the dynamics we want to explain are of aggregate variables, individual heterogeneity in payment methods will not affect them beyond getting the transaction shares right.

### 3 Idiosyncratic Uncertainty and Nominal Dynamics

In this section, we demonstrate analytically that there are at least three ways in which idiosyncratic shocks to cash-good preferences can improve the quantitative performance of the model. First, the dynamic behavior of the value of money and prices varies significantly with the probability that the marginal dollar is spent, i.e. that the cash constraint binds. As a result, the model with idiosyncratic shocks can accommodate values of the relative risk aversion (RRA) parameter in the standard RBC calibration range ( $\sigma \in [1, 4]$ ), whereas the model without shocks would require  $\sigma < 1$  to produce realistic dynamics of prices. Second, part of velocity fluctuation is now generated in the cash market, thus increasing the overall magnitude of velocity volatility, and velocity now depends in an intuitive way on nominal interest rates. Third, the standard general-equilibrium substitution channel in cash-credit good models between cash and credit good consumption is now dampened, because cash consumption will now only adjust for the binding realization of the shock. Most proofs of our results in this section are in appendix B.

#### 3.1 Dynamic Behavior of Real Balances

The dynamic behavior of money will be an essential input for relating velocity to interest rates. It is also, however, empirically relevant in itself: one uncontroversial empirical regularity is the degree

of persistence of interest rates, prices, and real balances, before and after detrending, over the business cycle. Nominal interest rates have an autocorrelation at quarterly frequency of 0.932; for real balances, it is 0.951.<sup>11</sup> It seems a minimal requirement that a monetary business cycle model can replicate the sign of these autocorrelations. This requirement turns out to have important implications for the range of the RRA parameter admissible in calibration.

For the sake of exposition, assume two preference shock realizations  $\vartheta_i$ , where  $\vartheta_h$  leads to a binding cash constraint, and  $\vartheta_l$  to a nonbinding constraint.<sup>12</sup> We write  $p$  for the probability of the high shock  $\vartheta_h$ . Note that if we set  $p = 1$ , we are back to the case with no idiosyncratic shocks. We temporarily simplify the utility function in the credit market to be fully linear,  $U(c) = c$ . To study the dynamics of real balances, we rework equilibrium conditions (29) to derive the relationship between real balances today  $\phi$  and expected real balances tomorrow  $\widetilde{\mathbb{E}\phi'} \equiv \mathbb{E} \left[ \frac{\phi'(S')}{1+\varpi} \frac{U'(c')}{U'(c)} \right]$ :

$$\phi = p\vartheta_h u'(\beta\widetilde{\mathbb{E}\phi'})\beta\widetilde{\mathbb{E}\phi'} + (1-p)\beta\widetilde{\mathbb{E}\phi'}, \quad (13)$$

This means that the real value of one unit of money, is a weighted averages of the value in use of the *marginal* unit of money, for which the need only arises with probability  $p$ , and the value when this unit not used, and carried over to the next period. Because of the normalization by the aggregate money stock, this equals aggregate real balances.

**Lemma 1.** *The elasticity of real balances today with respect to real balances tomorrow evaluated at an equilibrium  $\phi, \beta\widetilde{\mathbb{E}\phi'}$ , is given by*

$$\varepsilon_{\phi, \beta\widetilde{\mathbb{E}\phi'}} = \left(1 - \frac{1-p}{1+i}\right) (1-\sigma) + \frac{1-p}{1+i}. \quad (14)$$

Because of the positive second term on the RHS of (14), the elasticity of real balances today with respect to real balances tomorrow is brought closer to 1, and hence the demand for money is smoothened.<sup>13</sup> Real balances tomorrow in part determine directly real balances today, because with probability  $(1-p)$  the marginal unit of money is not used. Consider the case without idiosyncratic shocks, where the cash constraint always binds ( $p = 1$ ). The value of the marginal unit of money when it needs to spent, depends on the households' aversion to intertemporally varying consumption, captured by the intertemporal elasticity of substitution (i.e.s). Lower real balances tomorrow imply that the value of one unit of money is less in today's cash market, but less goods overall can be bought, raising the marginal utility. To be precise, when the cash constraint

<sup>11</sup>BP-filtered, nominal interest rate from three-month treasury bonds, real money balances from M2 and the GDP deflator (source: FRED2).

<sup>12</sup>In general, as  $i$  increases, more of the shock realizations may lead to a binding cash constraint; here, for exposition, we assume that only the high shock binds throughout. Generically, for small enough fluctuations in  $i$ , this assumption will hold. We will relax it in computation.

<sup>13</sup>Real balances today and tomorrow are measured here in the same units.

binds,  $q$  equals  $\beta \mathbb{E} \phi'$ ; this implies that the value of bringing one more unit of money when it is used to purchase cash consumption, is  $u'(\beta \mathbb{E} \phi') \beta \mathbb{E} \phi'$ . With an elasticity of substitution between zero and one, i.e.  $\sigma > 1$ , households are adverse enough to variation in consumption that, when real balances tomorrow are expected to decrease, the marginal utility rises so much that the value of money and hence, real balances today will increase, leading to a negative autocorrelation of real balances. To avoid such counterfactual behavior of monetary variables in the setting without idiosyncratic shocks, one has to make households less averse to consumption fluctuations ( $0 < \sigma < 1$ ). This, however, will imply an intertemporal elasticity of substitution that is nonstandard in business cycle modeling, leading to too strong responses of real variables to real shocks, e.g. technology shocks.

With idiosyncratic preference shocks, one does not have to make this unattractive choice between counterfactual monetary outcomes, or counterfactual outcomes on the real side. The proportional response of real balances today to foreseen changes in real balances tomorrow, is once again an average of the response when the marginal unit of money is not used and when this unit is spent, weighted now by  $\frac{1-p}{1+i}$  and  $(1 - \frac{1-p}{1+i})$  respectively, as shown in lemma 1. This means that households can be more averse to fluctuations in cash good consumption (conditional on the realization of the idiosyncratic shock), while preserving the highly positive autocorrelation of real balances. Lower real balances tomorrow can raise the value of an extra unit of money when the household does face a binding cash constraint, but when it is sufficiently likely that an additional dollar is unused and carried over to the next period, this would only work to dampen fluctuations in real balances, rather than create negative autocorrelations.<sup>14</sup>

### 3.2 Dynamic Behavior of Velocity

Denote by  $C$  aggregate consumption within a period. The consumption and output velocities of money in the above example with two idiosyncratic shocks can be written as

$$V_c = \frac{PC}{M} = \frac{c}{\phi} + (1-p) \frac{q_l(1+i)}{\phi} + p \frac{q_h(1+i)}{\phi}$$

$$V_y = \frac{PY}{M} = \frac{(y - (1-p)q_l - pq_h)}{\phi} + (1-p) \frac{q_l(1+i)}{\phi} + p \frac{q_h(1+i)}{\phi}$$

Let us look at consumption velocity. Observe that, as in standard cash-in-advance and cash-credit-good models, the constrained part of the cash market always contributes 1 to the level of consumption velocity, and nothing to velocity fluctuations, because  $\frac{q_h(1+i)}{\phi} = 1$ . Thus, if  $p = 1$ , then all velocity movement has to come from the credit market - i.e. from  $c$  or  $\phi$ . Instead, in our

<sup>14</sup>In our calibration, we will have a nominal interest rate rule with persistence. In a setting with  $p = 1$  and  $\sigma > 1$ , persistence in the nominal interest rate could be achieved by alternating expansions and contractions of the money supply. Again, this would be counterfactual.

model, velocity fluctuations are also created in the cash market, thanks to the low shocks where the cash constraint does not bind.<sup>15</sup>

One can also see this by looking at marginal rates of substitution between cash and credit market consumption. The MRS for the binding and non-binding cases is

$$\frac{\vartheta_h u'(q_h)}{U'(c)} = 1 + \frac{i}{p} \quad (15)$$

$$\frac{\vartheta_h u'(q_h)}{U'(c)} = 1, \quad (16)$$

(which implies  $\mathbb{E}_\vartheta[\frac{\vartheta u'(q_h)}{U'(c)}] = 1 + i$ ). Without preference shocks ( $p = 1$ ), cash market consumption thus always depends on nominal interest rates, as in (15). Preference shocks add agents who are not constrained ( $p < 1$ ), and whose cash market consumption does not depend on  $i$ , as in (16). Having arrived in the cash market, money holdings are predetermined, and households trade off spending a dollar in the cash market versus spending it next credit market. Given competitive pricing by retail firms', this results in an undistorted consumption allocation of unconstrained agents. Unconstrained agents do not adjust their consumption in response to changes in  $i$ , but in response to price changes, they adjust their money spending, and hence they contribute to fluctuations in velocity. Constrained agents respond to price changes through consumption, but the total amount of money spent does not move.

This analysis also sheds light on the nature of the inflation tax, relative to the standard cash-credit good model. Without preference shocks, money is costly to hold whenever the nominal interest rate is positive, and the wedge in the MRS (15) affects everyone equally. With preference shocks, the MRS of the unconstrained agents implies that their allocation is optimal. Thus, in terms of allocations, the inflation tax is borne only by the constrained agents, who ex ante, given a positive cost of insurance against preference shocks, opt to insure themselves only partially against the realization of high shocks. For a given level of  $q$  supplied by retailers, one can see an additional distortion, because the marginal utility of constrained households is higher than that of the unconstrained households, whereas a planner constrained only by  $q$  would equate these marginal utilities across preference shock realizations, like in the no-shock case. For the constrained agents, the cost of inflation is higher than the average in the no-shock model. We will evaluate the quantitative implications of this in the Welfare section.

Returning to velocity dynamics, the elasticity of consumption velocity with respect to  $i$  can be

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<sup>15</sup>Models with variable search intensity also create velocity fluctuations in the cash market (Wang and Shi 2006). Standard search models with fixed match probabilities do not.

divided into credit-market and cash-market components:

$$\varepsilon_{V_c, 1+i} = s_c(\varepsilon_{c, 1+i} - \varepsilon_{\phi, 1+i}) + s_{\text{cash}, nb} \cdot \varepsilon_{\psi, 1+i}, \quad (17)$$

where  $s_c$  is the share of the credit good in total consumption expenditure, and  $s_{\text{cash}, nb}$  is the share of cash consumption under non-binding preference shocks.

**Lemma 2.** *Elasticity of the cash market price with respect to the interest rate is always positive, and is given by*

$$\varepsilon_{\psi, 1+i} = \varepsilon_{\frac{1+i}{\phi}, 1+i} = \frac{1+i}{\sigma(p+i)} > 0. \quad (18)$$

Thus, the less risk-averse the household is, or the smaller the probability of a binding constraint is, the more of the velocity fluctuations originate in the cash market, ceteris paribus.

To conclude our analytical discussion, we now incorporate fully the response of credit-good consumption  $c$  to changes in prices and interest rates. To do this, we drop the linearity assumption on  $U(c)$ ; this adds a general-equilibrium feedback effect linking nominal interest rates and velocity, through substitution between cash and credit goods. The only assumption that we need for analytical tractability here is that capital is constant; while this shuts down one equilibrium effect, it does not alter the other effects that we focus on.<sup>16</sup>

**Proposition 1.** *The implicit elasticity of consumption velocity with respect to the nominal interest rate, caused by a one-time fully anticipated money injection (in addition to the constant rate of money growth consistent with a given steady state level of  $1+i$ ), is*

$$\varepsilon_{V_c, 1+i} = s_c \left( \frac{1}{\sigma} \frac{1+i}{p+i} - 1 \right) + s_{\text{cash}, nb} \left( \frac{1+i}{p+i} \right) \left( \frac{1}{\varepsilon_{U'(c), q_h} + \sigma} \right) \quad (19)$$

This elasticity is an equilibrium object: it tells us how the nominal interest rate and velocity vary when both variables move as a result of a one-time fully anticipated money injection.

*Proof.* For this proof, we redefine next period's real balances as  $\widetilde{\mathbb{E}\phi'} \equiv \mathbb{E}U'(c')\phi'/(1+\varpi)$ . From the free entry condition  $U'(c)q = \beta\widetilde{\mathbb{E}\phi'}$ , we get the elasticity of cash consumption to the money injection for a constrained agent as  $-\varepsilon_{q_h, 1+\varpi} = (\varepsilon_{U'(c), q_h} + 1)^{-1}$ . Then, from (13),

$$\varepsilon_{U'(c)\phi, \beta\widetilde{\mathbb{E}\phi'}} = \frac{d \ln U'(c)\phi}{d \ln \beta\widetilde{\mathbb{E}\phi'}} = \left( \frac{p+i}{1+i}(1-\sigma) \right) \frac{1}{\varepsilon_{U'(c), q_h} + 1} + \frac{1-p}{1+i}. \quad (20)$$

This gives elasticity of consumption velocity with respect to future value of money as

$$\varepsilon_{V_c, \beta\widetilde{\mathbb{E}\phi'}} = s_c \left\{ \left[ \left( -\frac{1}{\sigma} + 1 \right) \varepsilon_{U'(c), q_h} - \frac{p+i}{1+i}(1-\sigma) \right] \frac{1}{\varepsilon_{U'(c), q_h} + 1} - \frac{1-p}{1+i} \right\} - s_{\text{cash}, nb} \frac{1}{\varepsilon_{U'(c), q_h} + 1}.$$

From this and (20), we find (19). A detailed proof is in appendix B.3.  $\square$

<sup>16</sup>Capital fluctuations remain an important ingredient of the computed model below.

Note that the only difference between (20) and (14) is the term  $(\varepsilon_{U'(c),q_h} + 1)^{-1}$ , which captures the general equilibrium feedback between  $q_h$  and  $c$ , taking into account the optimal labor supply decision. As less is sold in the retail market, less has to be produced in the labor market. This improves marginal productivity of labor and raises credit consumption. As before, in equation (20), if  $p = 1$ , we would need  $\sigma < 1$  to get a positive sign for the autocorrelations of prices, real money stock and interest rates, and again, this constraint is relaxed if  $p < 1$ .

In (19), we recognize the different channels through which idiosyncratic uncertainty works: (i) credit market effects through the leftmost term; (ii) cash market channel through the right-side  $(1+i)/(p+i)$  term; and (iii) the general equilibrium substitution channel, through  $\varepsilon_{U'(c),q_h}$ . We show these components graphically as a function of  $\sigma$ , in figure 1. We see that idiosyncratic shocks raise the elasticity of velocity with respect to interest rates dramatically, as signified by the vertical difference between the grey dashed and black dashed lines in the graph, and allow for a positive elasticity for a much larger range of  $\sigma$  (here presented with  $p = 0.5$ ). We also observe, in the difference between the top dotted line and the top solid line, that the general equilibrium effect is small, but works to raise the elasticity of velocity with respect to the nominal interest rate. Keeping the size of the cash market the same, and lowering  $p$ , it can be shown that the sensitivity of velocity to interest rates through the cash market channel is raised.

## 4 Calibration

The model period is a quarter. We choose a CES form of the utility function:

$$U(c, q) = \frac{\left( (\alpha c^\nu + (1 - \alpha) \vartheta q^\nu)^{\frac{1}{\nu}} \right)^{1 - \sigma}}{1 - \sigma} \quad (21)$$

We will calibrate the model both with this function, and with the separable form that we used in most of our analytical work, which is a special case of the CES form. In that case, we use  $U(c) = \frac{c^{1-\sigma}}{1-\sigma}$  and  $u(q) = \frac{x_1 q^{1-\sigma}}{1-\sigma}$ . The production function is Cobb-Douglas:  $f(k, h) = k^\theta h^{1-\theta}$ .

In the separable-utility case, we need to calibrate parameters  $\beta$ ,  $\sigma$ ,  $A$ ,  $\theta$  and  $\delta$ , which are standard, as well as  $x_1$  and the process for the idiosyncratic shock  $\vartheta$ . For the CES utility, we calibrate  $\alpha$ , the share of credit goods in the consumption mix, instead of  $x_1$ , and  $\nu$ , the parameter that guides elasticity of substitution between cash and credit goods. Finally, the parameters of the exogenous driving processes  $\{\xi\}$ , and standard deviations  $\sigma_{\varepsilon_1}$  and  $\sigma_{\varepsilon_2}$  have to be calibrated.

We will use the M2 measure of money supply to compute the nominal moments in the data. We make this choice to follow the literature, e.g. Hodrick et al (1991) and Wang and Shi (2006), among others. Another reason of using this aggregate is that it exhibits much more stationarity

over time than M1. But the most important reason for our choice is that in our micro data work, we think of liquid payment methods as not only cash, but also checks and debit cards, which implies inclusion in the monetary aggregate of checking accounts. In addition, we are describing precautionary money demand, so it is intuitive that the monetary aggregate should include savings accounts. A concern may arise that some money in M2 earns interest, which does not happen in our model. In response to this concern, we present quantitative results for a modified version of the model, in which we allow interest to be paid not only on bonds, but also on money that is carried over from period to period as savings by households. Appendix C presents this modification and the resulting equilibrium conditions.<sup>17</sup> Notice that in this modified model, we are taking an extreme view that the representative liquid savings earn interest; in the data, a significant portion of the precautionary balances are held in non-interest-paying forms like cash and checking accounts (see Telyukova (2011)), so we are likely overstating the share of interest-bearing liquid balances.

For this version of the models, we have to change the driving process for interest rates. Details are below. The calibration of all the model versions is in table 1.

#### 4.1 Preference and Production Parameters

We calibrate the preference and production parameters of the model as follows.  $\beta = 0.9901$  matches the annual capital-output ratio of 3.  $\sigma = 2$  is chosen within, and on the lower side of, the standard range of  $[1, 4]$  in the literature.  $A$  is chosen each time to match aggregate labor supply of 0.3. The capital share of output is measured in the data to give  $\theta = 0.36$ . Quarterly depreciation rate of 2%, consistent with estimates in the data, gives  $\delta = 0.02$ .

For the separable model, we calibrate the constant  $x_1$  to match the size of the retail (cash) market. The target is 75% of total consumption in the model, consistent with the aggregate fact, documented in Telyukova(2011), that roughly 75% of the total value of consumer transactions in 2001 took place using liquid payments methods - cash, checks, and debit cards. This number was quoted at 82% in 1986 in Wang and Shi (2006), based on a consumer survey. We remain close to the 2001 target.

In an alternative calibration, we chose  $x_1$  to target the average level of M2 output velocity ( $V_y = 1.897$ ) in our data sample (1984-2007, as detailed below). This results in the size of the cash market of 50% of total consumption. The model calibrated this way produced the same dynamic results, so we will only show them in section 5.3 for an appropriate comparison (table 7).

For the CES utility case, the parameter  $x_1$  is no longer used. The parameters  $\alpha$  and  $\nu$  have to be

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<sup>17</sup>We also tried the version of the model where interest is paid on all money, both within period and across periods; the model results look very similar, so we do not present them here.



estimated jointly, together with  $\sigma_{\vartheta}$ , in an SMM procedure. For  $\alpha$ , we target the size of the cash-good market, 75%. For the parameter  $\nu$ , we target the interest rate elasticity of the ratio of aggregate cash good consumption to aggregate credit good consumption. In order to construct this measure, we use the Consumer Expenditure Survey over the period 1980-2004.<sup>18</sup> Over the entire period, we break household nondurable consumption items into cash goods and credit goods first, using the definitions discussed below for the calibration of preference shocks.<sup>19</sup> Then using household weights, we aggregate individual cash and credit good consumption. For the desired elasticity, we regress the cash-to-credit log-filtered consumption ratio on the T-Bill rate. The resulting interest rate elasticity is -1.24. The model is able to hit the targets exactly in each calibration. In the model without interest-paying money,  $\nu = 0.39$  implies the elasticity of substitution between the two types of goods of 1.6.

## 4.2 Idiosyncratic Preference Shock Process

We pose the log of the preference shock to be i.i.d.  $N(0, \sigma_{\vartheta})$ .<sup>20</sup> We interpret our preference shocks as causing fluctuations in household liquid consumption *beyond* expected (e.g. seasonal or planned) fluctuations in the data. To calibrate the process for this shock, we follow Telyukova (2011), which estimates a similar, but persistent and monthly, process, by matching time series properties of survey data on liquid household expenditures. We use quarterly data from the Consumer Expenditure Survey (CEX), and restrict attention to the period 2000-2002. We thus bias the target against our model: before the mid-1990's, credit cards were not ubiquitous, so that many more goods could be considered cash goods, resulting in a higher volatility estimate.

The key measure that we need is the unpredictable component of volatility of cash-good consumption in the data. We take this component of volatility to reflect optimal responses by households to unexpected preference shocks.<sup>21</sup> We adopt this measure as a calibration target, and use SMM to estimate the standard deviation of the shock process  $\sigma_{\vartheta}$  such that standard deviation of cash-good consumption in the model matches the data target.

The process of this measurement is described in Telyukova (2011) in detail; here we recap the

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<sup>18</sup>We thank Dirk Krueger and Fabrizio Perri for providing us with the aggregated CEX data set for this period. All the expenditure components in their data set are deflated by the relevant component of the CPI.

<sup>19</sup>Clearly, in the 1980's "cash goods" in the data would have been a much broader group than the 2004 definition we use here. However, in the absence of survey data from that period detailing which goods were and were not cash goods, the decision of when and how to change the definition over time becomes arbitrary. Instead, we choose the conservative approach of keeping the definition constant.

<sup>20</sup>Because of quasilinearity and credit markets in our setting, a shock process of the form  $\vartheta' = \vartheta^{\rho} \varepsilon'$  with  $\rho > 0$  would not change aggregate implications of our model, as can be shown from the first-order conditions of the problem.

<sup>21</sup>The preference shocks reflect any situation from being locked out of one's house to a significant household repair that requires payment by cash or check, e.g. In these situations, not having the money to meet the expense is very costly, which is well captured by a parameter that shifts (marginal) utility.

essentials. The first step is to separate out cash goods in the CEX data. As our measure, based on the American Bankers Association's 2004 survey of consumer payment methods, we use the following group: food, alcohol, tobacco, rents, mortgages, utilities, household repairs, childcare expenses, other household operations, property taxes, insurance, public transportation, and health insurance. Even in 2004, consumers reported paying for these types of goods with liquid assets (primarily cash and check) in 90% or more of transactions. This proportion would clearly be higher over our longer period of inquiry, 1984-2007. This measure is also conservative, along some other dimensions, from the standpoint of measuring unexpected expenses. First, volatility of expenses could be driven by seasonality (e.g. Christmas gift shopping), and to control for that in part, we remove any expenses reported as gifts; below we also remove seasonality in our regression analysis. Second, the cash-good category excludes many situations that may be reflections of emergencies that require liquid payment, such as an emergency purchase of (or downpayment on) a durable to replace - rather than repair - a broken durable, such as a car or an appliance. Similarly, medical payments, which include co-pays or other out-of-pocket expenses, some of which are unpredictable and may require a liquid payment - are not included either; the decision here was driven by the fact that medical expenses may be payable by credit card today, even though historically this would not be the case. Thus, in measuring the volatility of cash-good consumption, using a lot of the "smooth" good categories while excluding many that may reflect other types of emergencies besides repairs, may understate the measurement of the uncertainty that households face, against which they may hold liquid assets.

Using the above definition of cash goods, we estimate the following fixed-effect model with AR(1) innovations on liquid-consumption series, in order to extract the idiosyncratic shock:

$$\begin{aligned}\log(c_{it}^{liq}) &= \beta \mathbf{X}_{it} + u_i + \varepsilon_{it} \\ \varepsilon_{it} &= \rho \varepsilon_{i,t-1} + \eta_{it}.\end{aligned}\tag{22}$$

The vector  $\mathbf{X}$  includes, depending on specification, household observables, such as age (a cubic), education, marital status, race, earnings, family size, homeownership status, as well as a set of month and year dummies. Several specifications including different sets of these observables all produced nearly identical results.  $u_i$  is the household fixed effect. The residual  $\varepsilon_{it}$  is the idiosyncratic component of liquid consumption, and it further consists of a persistent component and a transitory component. Since our preference shock is assumed to be i.i.d., we consider the autoregressive component above as predictable, and the innovation  $\eta_{it}$  as reflecting household response to the preference shocks. Table 2 presents the standard deviation of  $\eta_{it}$  based on our benchmark cash-good measure above, as well as two alternatives that exclude some of the more predictable

expense groups. We will take the benchmark standard deviation of 19.6% to do our estimation in the model, clearly the most conservative measure.

The estimate of the standard deviation of the log-preference shock that results from our SMM procedure is  $\sigma_\theta = 0.4045$  in the separable model, and 0.2042 in the CES model, where in the latter case, the parameter is estimated jointly with  $\alpha$  and  $\nu$ . The values adjust slightly if we incorporate interest payments on money savings. We discretize our i.i.d. shock with 5 discrete shock states using the Tauchen (1986) method, with shocks at maximum two standard deviations away from their mean.<sup>22</sup> Finally, to convince the reader that we do not overstate the amount of uncertainty in expenses through our shock calibration, we plot in figure 2 the steady-state distribution of the log of liquid consumption in the model, and compare it to the empirical distribution of the log-consumption residual ( $\eta_{it}$ ), with bins centered at the same states as in the model. It is key for the quantitative performance of the model that we capture the probability of binding shocks correctly; in our 5-state calibration, this is reflected in the top consumption state, as only the top shock binds in our calibrated equilibrium. From the figure, it is apparent that our calibration captures this probability accurately.

### 4.3 Aggregate Shock Processes

Finally, we calibrate technology and monetary policy shocks. We model these as two separate processes, as described above. For the model with no interest paid on money, we estimate in our data sample the following two regressions:

$$\begin{aligned} z_t &= \xi_{zz} z_{t-1} + \varepsilon_1 \\ \ln \left( \frac{1 + i_t}{1 + \bar{i}} \right) &= \xi_{ii} \ln \left( \frac{1 + i_{t-1}}{1 + \bar{i}} \right) + \xi_{i\pi} \ln \left( \frac{1 + \pi_{t-1}}{1 + \bar{\pi}} \right) + \xi_{iy} \ln \left( \frac{y_{t-1}}{\bar{y}} \right) + \varepsilon_2 \end{aligned}$$

$z_t$  is the Solow residual measured in the standard way, and we take out the linear trend from both the Solow residual and the output series. The variables with bars capture long-term averages of the respective variables in our sample period, 1984-2007, as is standard in estimating central banks' targets in policy rules. Our choice of years captures the period when the Federal Reserve is thought to have used (implicit) inflation targeting. Notice that our interest rate rule depends on endogenous variables. We choose the Federal Funds rate as the measure of interest rates.

For the model with interest-paying money, denote by  $R$  the ratio  $\frac{1+i^m}{1+i^b}$ , where  $i^m$  is the interest rate paid on money, and  $i^b$  is the interest rate on bonds. We change the second equation above to

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<sup>22</sup>We also computed the model with 11 discrete shock states, and found the results to be robust.

the following system, which we estimate by VAR, using M2 own interest rate for  $i^m$ :

$$\begin{aligned}\ln\left(\frac{1+i_t^b}{1+i^b}\right) &= \xi_{ii}\ln\left(\frac{1+i_{t-1}^b}{1+i^b}\right) + \xi_{i\pi}\ln\left(\frac{1+\pi_{t-1}}{1+\bar{\pi}}\right) + \xi_{iy}\ln\left(\frac{y_{t-1}}{\bar{y}}\right) + \varepsilon_2 \\ \ln\left(\frac{R_t}{\bar{R}}\right) &= \xi_{rr}\ln\left(\frac{R_{t-1}}{\bar{R}}\right) + \xi_{ri}\ln\left(\frac{1+i_{t-1}^b}{1+i^b}\right) + \xi_{r\pi}\ln\left(\frac{1+\pi_{t-1}}{1+\bar{\pi}}\right) + \xi_{ry}\ln\left(\frac{y_{t-1}}{\bar{y}}\right) + \varepsilon_3.\end{aligned}$$

## 5 Results

### 5.1 The Role of Precautionary Money Demand

To highlight the quantitative role of precautionary demand for money, we compute, in addition to the four versions of our model described above, a version of the separable benchmark with the preference shocks shut down. Specifically, in this version, we assign to everyone the highest preference shock with probability 1; this means that everyone's cash constraint always binds. We refer to this version of the model as the “no-shock model”; it closely replicates standard cash-credit good models with only aggregate risk. The computational method is in Appendix D.

Table 3 summarizes the dynamic properties of some key nominal variables. The first column of the table presents moment in the data. The second column presents the results for the no-shock model. The last four columns show the results in our model with separable and CES utility, with and without interest-paying money. Notice that we are not targeting any of our result moments in calibration, so that the model is left free in terms of its dynamic performance.

As is clear from the table, for any model version, precautionary motive for holding money makes a dramatic difference for the performance of the model: without it, the model is not able to capture almost any of the moments in the data, while introducing precautionary demand makes the model align quite successfully on nearly all of the dimensions listed. When we do not target mean output velocity, we underpredict the level of both velocity measures. In our model, as in other monetary business cycle models, money turns over only once a quarter, so it is not surprising that the level is not high enough. It is also not surprising that in the no-shock model, mean velocity is higher than in the model with shocks; when the cash constraint is always binding, the cash market contributes exactly 1 to velocity level every period. In the model with shocks, that contribution is less than 1 for all the non-binding shock realizations; all but 7% of the households do not spend all of their money, and hence contribute less than 1 to velocity. If we target the level of velocity instead, we are able to match the levels very well without affecting the rest of the moments presented here. (See table 7).

Focusing on the models without interest-bearing money, in terms of volatility of velocity, as our analytical results suggested, we do much better in the model with the preference shocks than

in the model without. Even with our relatively low risk aversion parameter (a parameter that needed to be high in both Hodrick et al (1991) and Wang and Shi (2006) to begin to get significant velocity volatility), the model with the preference uncertainty gets close to the data in terms of the volatility of velocity. For output velocity, the separable model with the preference shocks produces 40% higher volatility than the no-shock model; for consumption velocity, the benchmark produces 60 *times* the volatility of the no-shock model. As discussed in our theory section, the reason is that with the introduction of preference shocks, the cash market contributes significantly to volatility of velocity, whereas it would contribute nothing if the cash constraint were always binding. Notice also that in the model with precautionary demand, we get the proportion of consumption velocity to output velocity volatility right, while it is very far off target in the no-shock model. Since consumption velocity is the major component of output velocity, consumption expenditure being 75% of GDP, the no-shock model is an unsatisfying theory of velocity dynamics because these dynamics come from the wrong source in that model.

Due to the properties of our exogenous driving process, we overpredict output volatility slightly and equally in both the benchmark and the no-shock model. The latter, however, underpredicts volatility of velocity substantially, leading us to conclude that the overprediction of output volatility is immaterial in creating excess velocity volatility.

Continuing down the list, the model with preference shocks replicates most moments very well, and much better than the model without shocks. As we analyzed in the model section, we expect that for RRA parameter  $\sigma > 1$ , as it is here, the correlation of velocity with nominal interest rates, and its elasticity, will be negative in the no-shock model, counter to the data. The relevant rows in the table confirm this. Instead, with the preference shocks, the relationship between velocity and nominal interest rates has the correct sign and magnitude. The correlations of output with growth of output and consumption also flip signs relative to data if the model has no idiosyncratic preference risk; with the shocks, the signs are correct, and the magnitudes are close to the data for the most part.

The CES and separable models produce almost the same results with respect to the above moments; in the CES case, we even overpredict volatility of velocity by a bit. If we modify the model to add interest-bearing money, the results are generally robust, in that it is still clear that precautionary demand for money makes a crucial difference relative to the no-shock case. The presence of interest payment on money reduces model volatility of velocity, and elasticity of velocity with respect to the bond interest rate, particularly in the separable model. This is not surprising: for unconstrained households, consumption in the cash market now responds to the movement not of the bond interest rate, but to the ratio of bond-to-money interest rates, which

dampens the response of cash consumption. In addition, since, as we mentioned before, we are making an extreme assumption that all precautionary balances earn interest, our model likely presents the lower bound on volatility of velocity. Notice however, that with CES and interest-bearing money, we recover most of the volatility properties, as well as other dynamic properties, of the benchmark without interest-bearing money; in some dimensions, the model's performance even improves relative to data. We view this as a strong endorsement for the role of precautionary demand for money: not only does the model do well along the standard dimensions in the literature, where typically money is not modeled as interest-bearing and is calibrated to M2, but our model passes an even higher bar of introducing interest-bearing money.

Table 4 presents the dynamic behavior of real balances ( $M/P$ ), as well as the autocorrelations of velocity, inflation and real balances. These moments, all close to the data in the benchmark model, show that velocity volatility does not come from excessive volatility in the value of the money stock or from volatility at the wrong frequency. Note also that in the no-shock model, volatility of real balances is extremely low relative to the data, again implying that it is unable to reproduce dynamics of the sources of velocity fluctuations.

## 5.2 Some Other Aggregate Facts

We now assess the performance of our model according to an additional set of facts, listed by Cooley and Hansen (CH, 1995) as some of the more significant monetary features of business cycles. A first set of these facts is presented in table 5. The first five columns list the performance of our model against the data, while the last two show the comparable moments from the CH data sample and model. The facts are that velocity is procyclical, prices are countercyclical, and that correlation of output and inflation with the growth of money supply is negative, with the latter being small. We match these facts in the model and get the magnitudes of the correlations about right. The correlation of money growth and inflation is the only fact that is affected by introduction of interest-bearing money: the sign reverses, although in the CES case, the magnitude approaches zero. Even with this reversal, our model performs better than the CH model on all dimensions. Appendix E shows additional dynamic results from the model via cross-correlations of some real and nominal variables with output.

Finally, we want to highlight some aspects of the data that we are not so successful in capturing. As is clear from table 6, our model, as the previous models in its class, misses the liquidity effect, i.e. the negative correlation of nominal interest rates with money growth, and prices and inflation are too flexible relative to data. All the versions of our model get these moments wrong in a similar way. The CH cash-in-advance model is again presented as a point of comparison and similarly misses

these moments. In general, it is not surprising we miss the liquidity effect, since our model lacks any rigidity in price adjustment. Sluggish adjustment of prices, as for example in Alvarez, Atkeson and Edmond (2008), is difficult to incorporate without a mechanism like market segmentation. We do not target this mechanism and hence did not expect to get the liquidity effect right.

### 5.3 Pitfalls of Calibration without Precautionary Demand

If precautionary demand for money is omitted from the model, the standard practice of calibrating monetary models to aggregate money demand, which may have a significant precautionary component, would likely produce misleading parameter values and thus affect the quantitative performance of the model. To demonstrate this here, we perform the following test. We take, again, a version of our separable benchmark model with the preference shocks shut down, so that the cash constraint always binds. But instead of fixing the calibration to the benchmark, as we did above, we will now calibrate the model to two standard targets in the monetary literature: the expected value of velocity,  $\mathbb{E}(V_y)$ , and the elasticity of inverse velocity with respect to the nominal interest rate,  $-\varepsilon_{V,1+i}$ . This exercise produces different values for parameters  $x_1$  (0.51),  $A$  (3.1) and, most importantly, the RRA parameter  $\sigma$ . The model without preference shocks can only reproduce the monetary targets with the value of  $\sigma = 0.15$ .<sup>23</sup> This is not surprising: we showed analytically that the no-shock model cannot get the sign of the elasticity of velocity with respect to nominal interest rates right unless  $\sigma < 1$ .

The dynamic properties of this model are presented in table 7, where we compare our separable model, now calibrated to match  $\mathbb{E}(V_y)$ , with the no-shock model with the same target. Even when targeting  $\mathbb{E}(V)$  and  $\varepsilon_{V,1+i}$ , the no-shock model does badly along other nominal dimensions, even for the same moments of *consumption* velocity, and now the quantitative implications on the real side of the model suffer noticeably as well. For instance, here the no-shock model doubles volatility of output, consumption and investment relative to data, while the benchmark gets these standard deviation measures fairly close to the data. In other words, even if we were willing to accept the calibrated parameters that this exercise requires, the results that the model produces are far inferior to the performance of our benchmark with preference shocks. This is true along both nominal and real dimensions, even though the model is rigged, in how it is parameterized, to do well quantitatively on the nominal side. Modeling precautionary demand for money solves this problem.

With a CES utility, we can derive the value of the parameter  $\nu$  needed in order to match the

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<sup>23</sup>Values of  $\sigma$  very close to this commonly arise in monetary models without precautionary money demand, from sticky-price models (Rotemberg and Woodford, 1997) to monetary search models (Lagos and Wright, 2005).

target  $\varepsilon_{V_c, 1+i} = 4.15$ , as described in the above subsection. First, we can show that the elasticity of consumption velocity with respect to the nominal interest rate in this version of the model is given by  $\varepsilon_{V_c, 1+i} \approx s_{cr} \frac{c}{c+q} \frac{\nu}{1-\nu} \approx 4.15$ . With credit consumption  $c/(c+q)$  measuring at 47% of total consumption, as was the case to target the expected level of consumption velocity, this expression implies the parameter  $\nu = 0.915$ . If we target the credit consumption share of 25%, as in the SMM procedure described before, then we derive  $\nu = 0.94$ . These values of parameters imply elasticity of substitution between cash and credit goods of 11.8 and 20, respectively, for the CES model with no preference shocks to match the elasticity target.

But the model with no preference shocks also gives a closed-form implication for the parameter  $\nu$ , so that we can estimate it in the data directly. The model with CES utility implies the marginal rate of substitution between cash and credit goods given by  $\frac{1-\alpha}{\alpha} \left(\frac{q}{c}\right)^{\nu-1} = 1+i$ . Taking logs and rearranging, we can estimate  $\nu$  by running the following regression:

$$\log\left(\frac{q}{c}\right) = -\log\left(\frac{1-\alpha}{\alpha}\right) \frac{1}{\nu-1} + \frac{1}{\nu-1} \log(1+i).$$

Based on logged and detrended data that we used before, we get  $\nu = 0.85$ , which implies the elasticity of substitution between the two goods of 6.8. Thus the data targets once again prove impossible to match using the model with no precautionary demand.

All of this suggests that omitting precautionary money demand from monetary models may produce inaccurate results in not only matching data facts, but also conducting policy experiments and drawing normative conclusions.

## 6 Welfare Costs of Inflation

### 6.1 Inflation Level

As we discussed above, in a model without precautionary demand for money, all agents bear the inflation tax equally: it comes from the wedge in the marginal rate of substitution between the cash and credit good created by the nominal interest rate. In our model with idiosyncratic risk, changes in steady state inflation distort cash-good consumption only for the binding shock realizations, but the constrained households value this consumption more than average, and are therefore more sensitive to these distortions. Relative to a model without precautionary demand, the first channel would diminish the welfare cost of inflation, because not all agents bear it, while the second would increase it for the constrained agents. Comparing steady states, we calculate the welfare cost of inflation as the percentage reduction in consumption under the Friedman Rule that would make a household indifferent between the Friedman Rule and a higher-inflation state. This measure is



$1 - \Delta$ , with  $\pi_{FR}$  denoting inflation at the Friedman Rule:

$$U(\Delta c(\pi_{FR})) + \mathbb{E}[\vartheta u(\Delta q_{\vartheta}(\pi_{FR}))] - Ah(\pi_{FR}) = U(c(\pi)) + \mathbb{E}[\vartheta u(q_{\vartheta}(\pi))] - Ah(\pi),$$

We solve for  $\Delta$ , and derive the steady state quantities in closed form (see appendix F). The dashed (red) line in panel(a) of figure 3 is the welfare cost of inflation in the no-shock model; the solid (blue) line is the cost of welfare in the model with idiosyncratic shocks. First, the welfare cost of 10% yearly inflation, relative to the Friedman rule, is 0.2% of the efficient level of consumption in the no-shock model, but is more than twice that, 0.5%, in our benchmark model with idiosyncratic risk. These results are slightly lower than in the literature, because the standard welfare computation does not include the decrease in disutility of labor when cash-good consumption declines with inflation.<sup>24</sup>

Second, the difference between the two lines is increasing with the level of annual inflation. This is because not only the cost of inflation increases for a given constrained agent as inflation rises, but in addition, the proportion of constrained agents increases with inflation. At 100% inflation, where in our benchmark model close to 30% of households are constrained, compared to only 7% at inflation rates below 9%, the welfare cost in the no-shock model is 1.6%, while in the model with shocks, it is 2.8%. Thus, a model with a full distribution of idiosyncratic shocks allows us to give a better approximation of welfare costs at high levels of inflation than models in which the cash constraints always binds, and in which the share of consumption subject to this constraint is calibrated using velocity in low-inflation data. In our steady state comparison, the underestimate of welfare costs in the no-shock model is increasing over a large range of inflation (at least up to an annual inflation of 1200%), and is relatively stable afterwards.

## 6.2 Inflation Uncertainty

In the benchmark model, there is no uncertainty about inflation within a period: the current aggregate state is revealed at the beginning of the period, households make their decisions on how much money to hold, and firms set supply of the cash good, knowing the value of money, and hence the cash-good price, in advance. We can modify the timing of the model, however, to introduce inflation “surprises”, so that households and firms make their credit-market decisions before they know the second-subperiod price level. In our setting, we modify the benchmark to allow households to find out the aggregate state of the economy for period  $t + 1$  at the start of subperiod 2 of  $t$ . The information structure in this model is meant to be comparable to Svensson

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<sup>24</sup>Without accounting for the labor disutility, the welfare cost of 10% inflation in the model with shocks, compared with the Friedman rule, is 1.58% of the efficient level of consumption, and about 1.2% of output. These numbers are 0.5% in Cooley and Hansen (1989), and 1.5% Lucas (2000) and 1.6% in Lagos and Wright (2005)

(1985); the details are in appendix G.

This timing adjustment adds a distortion from increases in inflation, because the cash-good price  $\psi_t$  will adjust mid-period to the information regarding next period's aggregate state. Now retail firms decide on how much good  $q_t$  to take out of the credit market, and households choose money holdings, before they know  $\psi_t$ . Suppose that households observe that inflation will be higher than expected, implying lower value of money  $\phi_{t+1}$ . The constrained households cannot increase their cash-good consumption, and can buy less in the high-shock state than they expected before. However, the supply of the cash good is fixed from the credit market, and the shortfall of demand from constrained households has to be made up by demand from unconstrained households. This means that relative to a case without unexpected inflation, the MRS of a constrained household will increase as its consumption will decline, while that of an unconstrained household will be lower than 1, so the distortion between the two types of households is exacerbated by unexpected inflation changes. Thus, an *unexpected* increase in inflation would decrease welfare by a larger amount than an *unexpected* increase, because  $q$  cannot adjust. As a result, a mean-preserving increase in the variance of inflation shocks can lower ex-ante welfare in this version of the model, whereas it would have no impact in the model with no inflation surprises. As panel (b) of figure 3 shows, welfare cost of inflation uncertainty accelerates as the standard deviation of log-inflation increases; the welfare cost of 10% standard deviation of inflation is just below 2.5% of consumption.

## 7 Conclusion

In this paper, we study the aggregate implications of precautionary demand for money. We highlight the importance of modeling uninsurable idiosyncratic risk in preferences as a cause of precautionary motive for holding liquidity. By incorporating this idiosyncratic risk into a standard monetary model with aggregate risk, and by carefully calibrating the idiosyncratic shocks to data, we find that the model matches many dynamic moments of nominal variables well, and greatly improves on the performance of existing monetary models that do not incorporate such idiosyncratic shocks. We show also that omitting precautionary demand while targeting, in calibration, data properties of money demand – a standard calibration practice – produces inferior performance in terms of matching the data, potentially misleading implications for parameters of the model, and an understatement of welfare costs of inflation, and may therefore adversely affect the model's policy implications as well.

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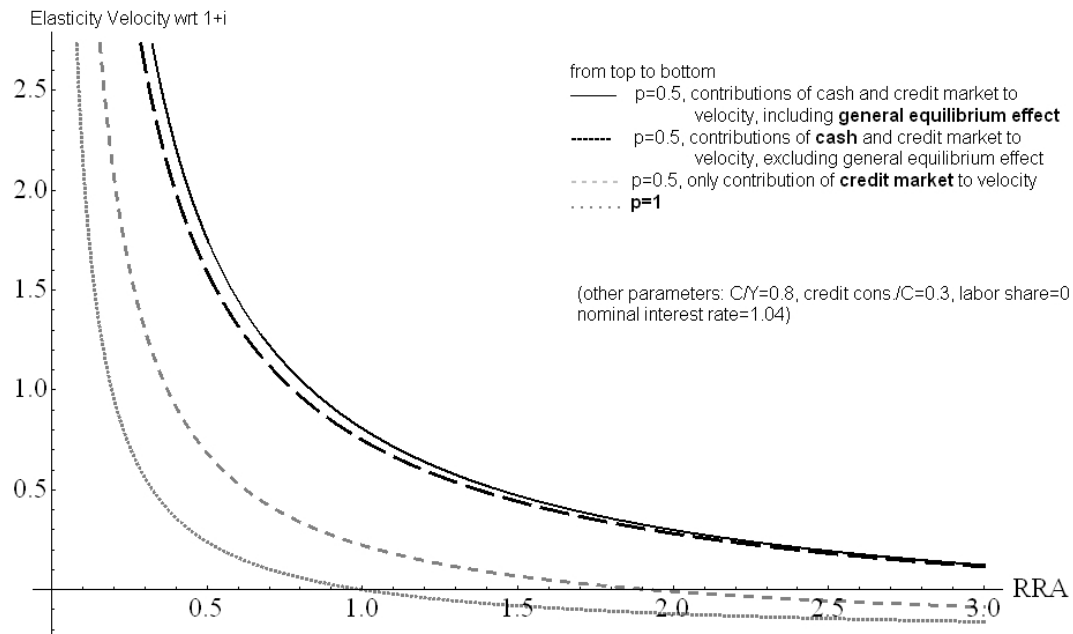


Figure 1: Contributions of Idiosyncratic Uncertainty to Interest Rate Elasticity of Velocity

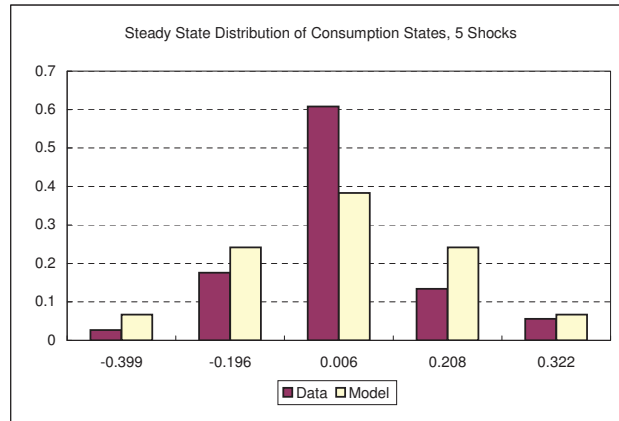
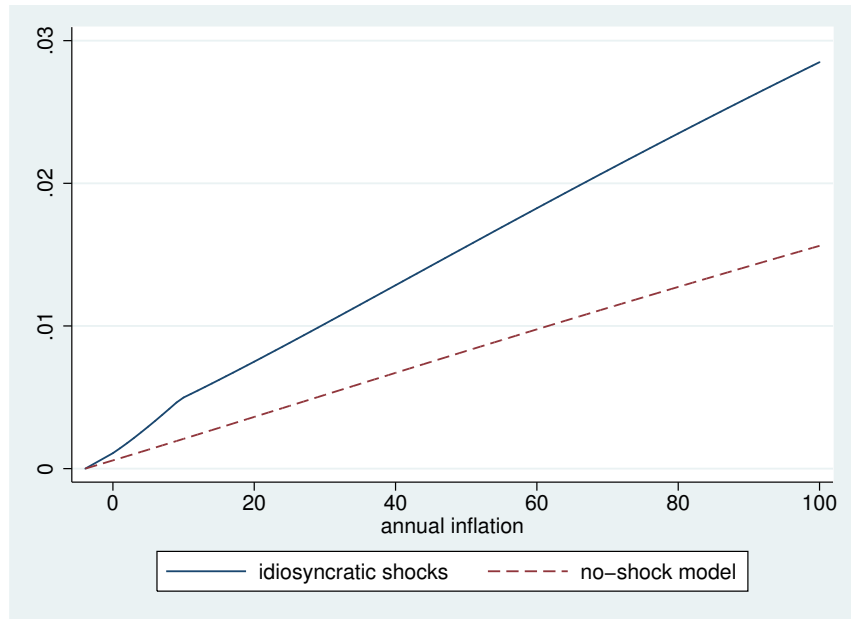
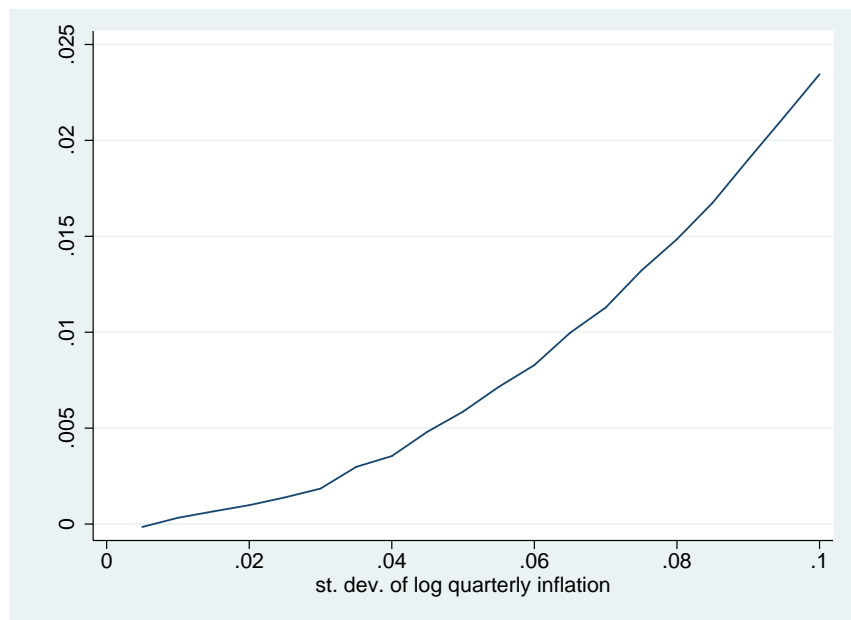


Figure 2: Distribution of Log-Consumption of Cash Goods, Data vs. Model Discretization



(a) Inflation Level



(b) Inflation Uncertainty

Figure 3: Welfare Cost of Inflation



Table 1: Calibrated Parameters

	Separable	CES	Separable with $i^m$	CES with $i^m$
$\beta$	0.99	0.99	0.99	0.99
$\sigma$	2	2	2	2
$A$	34	5	34	5
$x_1$	6	—	6.33	—
$\alpha$	—	0.33	—	0.33
$\nu$	—	0.39	—	0.43
$\theta$	0.36	0.36	0.36	0.36
$\delta$	0.02	0.02	0.02	0.02
$\sigma_\vartheta$	0.41	0.20	0.39	0.21
$\xi_{zz}$	0.94	0.94	0.94	0.94
$\xi_{ii}$	0.78	0.78	0.78	0.78
$\xi_{i\pi}$	0.13	0.13	0.12	0.12
$\xi_{iy}$	0.01	0.01	0.01	0.01
$\xi_{rr}$	—	—	0.89	0.89
$\xi_{rr}$	—	—	0.01	0.01
$\xi_{ry}$	—	—	-0.006	-0.006
$\xi_{r\pi}$	—	—	-0.05	-0.05
$\sigma_{\varepsilon_1}$	0.006	0.006	0.006	0.006
$\sigma_{\varepsilon_2}$	0.001	0.001	0.001	0.001
$\sigma_{\varepsilon_3}$	—	—	0.0007	0.0007
$\sigma_{\varepsilon_2, \varepsilon_3}$	—	—	$-7 \times 10^{-7}$	$-7 \times 10^{-7}$

“Separable” – benchmark model with separable utility; “CES” – benchmark with CES utility. “With  $i^m$ ” is the version of the model with interest-bearing money.

Table 2: Unpredictable Volatility of Liquid Consumption, Quarterly CEX Data

	Standard Deviation of $\eta_{it}$ (%)
Benchmark	19.6
Excluding food	27.5
Excluding food and property taxes	29.4

Standard deviation, converted into percent, of the transitory component of the residual of the regression of log-cash good consumption on household characteristics, CEX 2000-2002. See equation (22).

Table 3: Dynamic Properties of the Model

Moment	Data	No-Shock Model	Separable	CES	Separable with $i^m$	CES with $i^m$
$\mathbb{E}(V_y)$	1.897	1.812	1.357	1.339	1.293	1.248
$\mathbb{E}(V_c)$	1.120	1.380	1.033	1.020	0.984	0.950
$\sigma(V_y)$	0.017	0.010	0.014	0.021	0.011	0.013
$\sigma(V_c)$	0.014	0.0002	0.012	0.019	0.006	0.009
$\sigma(y)$	0.009	0.012	0.012	0.012	0.012	0.012
$\sigma(1 + i^b)$	0.0026	0.001	0.002	0.0025	0.002	0.002
$corr(V_y, y)$	0.638	0.993	0.585	0.386	0.817	0.684
$corr(V_y, g_y)$	0.059	0.289	0.142	0.078	0.217	0.159
$corr(V_c, g_y)$	-0.094	0.110	-0.071	-0.065	-0.074	-0.084
$corr(V_y, g_c)$	0.127	0.539	0.233	0.109	0.242	0.464
$corr(V_c, g_c)$	-0.027	0.176	-0.155	-0.148	-0.128	-0.134
$corr(V_y, 1 + i^b)$	0.714	-0.210	0.645	0.558	0.390	0.585
$corr(V_c, 1 + i^b)$	0.690	-0.896	0.897	0.648	0.912	0.915
$\varepsilon_{V_y, 1+i^b}$	5.072	-1.747	4.546	4.744	2.542	4.115
$\varepsilon_{V_c, 1+i^b}$	4.158	-0.123	5.072	5.046	3.363	4.653
$corr(1 + \pi, 1 + i^b)$	0.529	0.768	0.361	-0.032	0.572	0.465

All data logged and BP filtered. Data period: 1984-2007. Velocity moments calculated based on M2. Bond interest rate is the Fed Funds rate. Inflation measured based on GDP deflator.  $g_y$  refers to output growth. "Separable" - benchmark model with separable utility; "CES" - benchmark with CES utility. "With  $i^m$ " is the version of the model with interest-bearing money. "No-Shock" model is the version of the separable model with idiosyncratic preference shocks shut down.

Table 4: Properties of Velocity Volatility

Moment	Data	No-Shock Model	Separable	CES	Separable with $i^m$	CES with $i^m$
$\sigma(M/P)$	0.013	0.003	0.012	0.018	0.007	0.010
$\text{corr}(V_y, V_{y,-1})$	0.941	0.898	0.898	0.902	0.901	0.903
$\text{corr}(V_c, V_{c,-1})$	0.937	0.896	0.898	0.902	0.911	0.911
$\text{corr}(\pi, \pi_{-1})$	0.901	0.870	0.844	0.840	0.864	0.851
$\text{corr}(M/P, M/P_{-1})$	0.944	0.921	0.898	0.901	0.908	0.908

All data logged and BP filtered. Data period: 1984-2007. Velocity and money supply moments calculated based on M2. Inflation measured based on GDP deflator. "Separable" – benchmark model with separable utility; "CES" - benchmark with CES utility. "With  $i^m$ " is the version of the model with interest-bearing money. "No-Shock" model is the version of the separable model with idiosyncratic preference shocks shut down.

Table 5: More Monetary Business Cycle Facts

Moment	Data	Sep.	CES	Sep. + $i^m$	CES + $i^m$	CH data	CH
$corr(V, y)$	0.64	0.58	0.39	0.82	0.68	0.37	0.948
$corr(p, y)$	-0.13	-0.28	-0.10	-0.26	-0.28	-0.57	-0.22
$corr(g_m, y)$	-0.11	-0.15	-0.10	-0.27	-0.25	-0.12	-0.01
$corr(g_m, \pi)$	-0.06	-0.13	-0.40	0.38	0.07	-0.29	0.92

All data logged and BP filtered. Data period: 1984-2007. Velocity moments calculated based on M2. Inflation and prices measured based on GDP deflator.  $g_m$  refers to money supply growth. "Separable" - benchmark model with separable utility; "CES" - benchmark with CES utility. "With  $i^m$ " is the version of the model with interest-bearing money.

Table 6: Liquidity Effect

Moment	Data	Separable Model	CH data	CH Model
$corr(g_m, i)$	-0.7(M1)/0.07(M2)	0.79	-0.27	0.72
$corr(y, i)$	0.54	-0.13	0.40	-0.01
$corr(y, \pi)$	0.37	-0.25	0.34	-0.14
$corr(g_m, p)$	0.03	0.61	-0.16	0.43

All data logged and BP filtered. Data period: 1984-2007. Money supply moments calculated based on M2. Bond interest rate is the Fed Funds rate. Inflation measured based on GDP deflator.  $g_m$  refers to money supply growth. "Separable" – benchmark model with separable utility.

Table 7: No-Shock Model Targeting Money Demand in the Data, Separable Utility

Moment	Data	Separable (Target $\mathbb{E}(V_y)$ )	No-Shock Model (Target Money Demand)
$\mathbb{E}(V_y)$	<b>1.897</b>	<b>1.898</b>	<b>1.895</b>
$\mathbb{E}(V_c)$	1.120	1.445	1.442
$\sigma(V_y)$	0.017	0.014	0.028
$\sigma(V_c)$	0.014	0.011	0.003
$\sigma(y)$	0.009	0.012	0.022
$\sigma(c)$	0.005	0.003	0.012
$\sigma(inv)$	0.050	0.044	0.114
$\sigma(1+i)$	0.0026	0.002	0.002
$corr(V_y, y)$	0.638	0.602	0.905
$corr(V_y, g_y)$	0.059	0.145	0.411
$corr(V_c, g_y)$	-0.094	-0.070	-0.185
$corr(V_y, g_c)$	0.127	0.262	0.323
$corr(V_c, g_c)$	-0.027	-0.139	0.256
$corr(V_y, 1+i)$	0.714	0.638	0.333
$corr(V_c, 1+i)$	0.690	0.897	0.999
$\varepsilon_{V_y, 1+i}$	<b>5.072</b>	4.469	<b>5.030</b>
$\varepsilon_{V_c, 1+i}$	4.158	4.994	1.763
$corr(1+\pi, 1+i)$	0.529	0.358	0.657
RRA $\sigma$		2.0	0.15

All data logged and BP filtered. Data period: 1984-2007. Velocity moments calculated based on M2. Bond interest rate is the Fed Funds rate. Inflation measured based on GDP deflator.  $g_y$  refers to output growth. Benchmark: separable model, calibrated to target  $\mathbb{E}(V_y)$ . “No-Shock” model is the version of the separable model with idiosyncratic preference shocks shut down. Bolded quantities represent calibration targets. RRA  $\sigma$  is the value of the relative risk aversion parameter needed in calibration in order to match the targets.

## A APPENDIX: Analysis of Household Problem

### A.1 Walrasian Market creates Homogeneity

For general utility functions, different realizations of the idiosyncratic preference shock would lead to a nontrivial distribution of wealth (with, for example, those who have recently experienced a sequence of high  $\vartheta$ s now being poorer on average). In turn, households with different wealth could make different portfolio decisions, and hence the distribution across individual state variables would be relevant for equilibrium prices.

However, the quasi-linear specification of the problem allows equilibria in which all heterogeneity created in the second subperiod washes out in the credit market.<sup>25</sup> This occurs if the boundary conditions of  $h$  are never hit, which we assume to be the relevant case below. Our quantitative strategy later is to solve the problem assuming that the optimal choice of  $h$  is interior, and check in our calibrated equilibrium whether this is indeed the case.

After substituting the budget constraint for  $h$  into the household's value function, we can split the value function in two parts:

$$V(s, S) = A \left( \frac{\phi m + (1 + r - \delta)k + \phi b(1 + i_{-1})}{w} \right) + \max_{\dots} \left\{ U(c) - A \left( \frac{c + \phi \tilde{m} + k' + \phi b'}{w} \right) + \mathbb{E}_{\vartheta} [\vartheta u(q_{\vartheta}) + \beta \mathbb{E}_{z', i'} [V(s', S')]] \right\}. \quad (23)$$

From this, Result 1 is immediate, under the assumption of interiority on  $h$ .

Household wealth differs at the beginning of the Walrasian market, due to heterogeneous trading histories in the previous cash market, but households adjust their hours worked to be able to get the same amount of  $c, \tilde{m}, k', b'$ . The value function  $V(\cdot)$  is differentiable in  $k, m, b$ , and the envelope conditions are independent of the individual state variables. Hence, the expectation over  $\vartheta$  does not matter for intertemporal choice variables, for example:

$$\mathbb{E}_{\vartheta} [\mathbb{E}_{z', i'} V_m(s, S)] = \mathbb{E}_{z', i'} V_m(s, S) = \mathbb{E}_{z', i'} \left[ \frac{A \phi(S)}{w(S)} \right].$$

The problem is weakly concave in capital, labor and bond holdings, and the solution is interior, as long as  $h$  is interior. The first-order condition with respect to credit-good consumption, and the Euler equations with respect to capital and bonds, thus look as follows:

---

<sup>25</sup>This result has been used extensively in models that combine Walrasian markets with bilateral trade and idiosyncratic *matching* risk, such as Lagos and Wright (2005) and Rocheteau and Wright (2005). Here we use it to combine Walrasian markets with cash markets and idiosyncratic *preference* risk.



$$U'(c(S)) = \frac{A}{w(S)} \quad (24)$$

$$U'(c(S)) = \beta \mathbb{E}_{z', i'} [U'(c(S'))(1 + r(S') - \delta)] \quad (25)$$

$$\phi U'(c(S)) = \beta \mathbb{E}_{z', i'} \left[ \frac{\phi'}{1 + \varpi} U'(c(S')) \right] (1 + i) \quad (26)$$

For future reference, we introduce the following notation, using marginal utilities defined in terms of the marginal productivity of labor (24):

$$\mathbb{E} \left[ \frac{U'(c')}{U'(c)} \right] \equiv \tilde{\mathbb{E}}, \quad \mathbb{E} \left[ \frac{\phi'(S')}{1 + \varpi} \frac{U'(c')}{U'(c)} \right] \equiv \widetilde{\mathbb{E}\phi'}.$$

## A.2 The Choice of Money Holdings and Cash Market Consumption

Denote by  $\mathbb{P}(\vartheta)$  the probability of a particular shock  $\vartheta$  occurring. Taking as given (25)-(26), the first-order conditions with respect to  $\tilde{m}$  and  $q_\vartheta$  give

$$\mathbb{P}(\vartheta) \left( \frac{\vartheta u'(q_\vartheta)}{U'(c)} - 1 \right) - \psi \frac{\mu_\vartheta}{U'(c)} = 0 \quad (27)$$

$$-\phi + \sum_{\vartheta} \frac{\mu_\vartheta}{U'(c)} + \beta \widetilde{\mathbb{E}\phi'} = 0, \quad (28)$$

with the appropriate complementary slackness conditions (see equilibrium conditions below). It is immediate that in this model cash balances are not spent in full for realizations of  $\vartheta$  that are low enough. Since a social planner would equate  $U'(c)$  to  $\vartheta u'(q_\vartheta)$ , the following conclusions can be drawn:

**Result 2.** *If a shock  $\vartheta$  results in a nonbinding constraint, then  $q_\vartheta$  is the efficient quantity. Moreover, as long as the cash constraint does not bind, the quantity  $q_\vartheta$  does not respond to the interest rate.*

Moreover, also observe that if some  $\hat{\vartheta}$  leads to a binding constraint, then for every  $\vartheta > \hat{\vartheta}$ , the cash constraint will bind. If  $\hat{\vartheta}$  leads to a slack cash constraint, any  $\vartheta < \hat{\vartheta}$  will lead to a nonbinding constraint. A binding cash constraint leads to underconsumption of the cash good relative to the social optimum.

## A.3 Equilibrium Conditions

We summarize the above discussion in the system of first-order conditions and Euler equations that characterize the equilibrium of the problem:

$$\begin{aligned}
U'(c) &= \beta \mathbb{E}[U'(c')(1 + r' - \delta)] \\
U'(c) &= \frac{A}{w} \\
\psi &= \frac{1+i}{\phi} \\
\frac{\mu_{\vartheta}}{U'(c)} &= \mathbb{P}(\vartheta) \left( \frac{\vartheta u'(q_{\vartheta})}{U'(c)\psi} - \frac{\phi}{1+i} \right); \mu_{\vartheta}(\tilde{m} - \psi q_{\vartheta}) = 0 \quad \forall \vartheta \\
\phi &= \frac{\sum_{\vartheta} \mu_{\vartheta}}{U'(c)} + \frac{\phi}{1+i} \\
\frac{\phi}{1+i} &= \beta \widetilde{\mathbb{E}\phi'} \\
y + (1 - \delta)k &= c + k' + \sum_{\vartheta} \mathbb{P}(\vartheta) q_{\vartheta} \\
z' &= \xi_{zz} z + \varepsilon'_1 \\
\widetilde{(1+i')} &= \xi_{ii} \widetilde{(1+i)} + \xi_{i\pi} \tilde{\pi} + \xi_{iy} \tilde{y} + \varepsilon'_2
\end{aligned} \tag{29}$$

## B Proofs of Analytical Results: Idiosyncratic Uncertainty and Nominal Dynamics

### B.1 Lemma 1

*Proof.* The derivative of  $\phi$  with respect to  $\beta\mathbb{E}\phi'$ , using (13), (26), the cash-market money constraint and  $\tilde{m} = 1$ , is

$$\frac{d\phi}{d(\beta\mathbb{E}\phi')} = p\vartheta_h(u''(\beta\mathbb{E}\phi')\beta\mathbb{E}\phi' + u'(\beta\mathbb{E}\phi')) + (1-p)$$

Divide both sides by  $\phi/(\beta\mathbb{E}\phi')$ , and using (13), we find

$$\varepsilon_{\phi, \beta\mathbb{E}\phi'} = p \frac{\vartheta_h(u''(\beta\mathbb{E}\phi')\beta\mathbb{E}\phi' + u'(\beta\mathbb{E}\phi'))}{p\vartheta_h u'(\beta\mathbb{E}\phi') + (1-p)} + (1-p) \frac{\beta\mathbb{E}\phi'}{\phi}.$$

Rewriting this as a function of the interest rate  $(\phi/\beta\mathbb{E}\phi')$ , this elasticity then becomes equation (14).  $\square$

### B.2 Lemma 2

*Proof.* One can derive that  $\varepsilon_{\frac{1+i}{\phi}, 1+i} = 1 - \varepsilon_{\phi, 1+i}$ . Substituting in

$$\varepsilon_{\phi, 1+i} = \frac{1}{\varepsilon_{1+i, \phi}} = \frac{1}{1 - \varepsilon_{\beta\mathbb{E}\phi', \phi}} = \frac{\varepsilon_{\phi, \beta\mathbb{E}\phi'}}{\varepsilon_{\phi, \beta\mathbb{E}\phi'} - 1}, \quad (30)$$

we find

$$\varepsilon_{\psi, 1+i} = \varepsilon_{\frac{1+i}{\phi}, 1+i} = 1 - \varepsilon_{\phi, 1+i} = \frac{-1}{\varepsilon_{\phi, \beta\mathbb{E}\phi'} - 1}$$

Putting the last equation together with (14) yields (18).  $\square$

### B.3 Proposition 1.

**Proposition 1.** *The implicit elasticity of consumption velocity with respect to the nominal interest rate, caused by a one-time fully anticipated money injection (in addition to the constant rate of money injection consistent with a given steady state level of  $1+i$ ), is*

$$\varepsilon_{V_c, 1+i} = s_c \left( \frac{1}{\sigma} \frac{1+i}{p+i} - 1 \right) + s_{cash, nb} \left( \frac{1+i}{p+i} \right) \left( \frac{1}{\varepsilon_{U'(c), q_h} + \sigma} \right) \quad (31)$$

*Proof.* The elasticity of velocity with respect to a change in the interest rate (caused by a one-time anticipated additional injection of money  $1+\varpi$ ) is

$$\varepsilon_{V_c, 1+i} = \frac{d \ln V_c}{d \ln 1 + \varpi} \bigg/ \frac{d \ln 1 + i}{d \ln 1 + \varpi}$$

Velocity is given by

$$V_c = \frac{PC}{M} = \frac{c}{\phi} + (1-p) \frac{q_l(1+i)}{\phi} + p \frac{q_h(1+i)}{\phi},$$

hence,

$$\varepsilon_{V_c, 1+\varpi} = s_c(\varepsilon_{c, 1+\varpi} - \varepsilon_{\phi, 1+\varpi}) - s_{cash, nb} \varepsilon_{q_h, 1+\varpi},$$

using  $\varepsilon_{\psi, 1+\varpi} = -\varepsilon_{q_h, 1+\varpi}$ .

Since a one-time fully anticipated money injection does not affect tomorrow's  $\phi$  or  $U'(c')$ , we formulate the elasticities in terms of  $\widetilde{\mathbb{E}\phi'}$ , which for the duration of the proof we define as  $\widetilde{\mathbb{E}\phi'} \stackrel{def}{=} U'(c')\phi'/(1+\varpi)$ ; then  $\varepsilon_{1+\varpi, \beta\widetilde{\mathbb{E}\phi'}} = -1$ , and

$$\varepsilon_{V_c, 1+i} = \varepsilon_{V_c, \widetilde{\mathbb{E}\phi'}} / \varepsilon_{1+i, \widetilde{\mathbb{E}\phi'}}.$$

To derive  $\varepsilon_{c, \widetilde{\mathbb{E}\phi'}}$ , let us derive how  $h$  varies with  $c$  in the equilibrium. From equation (24), the elasticity is

$$\varepsilon_{c, h} = -\frac{\alpha}{\sigma}, \quad \varepsilon_{U'(c), h} = \alpha \quad (32)$$

Now, from the household budget constraint, we use (32) to derive

$$\varepsilon_{c, q_h} = -\frac{s_{q_h}}{s_c + \frac{1-\alpha}{\alpha}\sigma}, \quad \varepsilon_{U'(c), q_h} = \frac{s_{q_h}\sigma}{s_c + \frac{1-\alpha}{\alpha}\sigma} > 0, \quad (33)$$

where  $s_{q_h}$  is the share of total output going to  $q_h$  consumption,  $s_{q_h} = (pq_h)/Y$ ; likewise  $s_c = c/Y$ . Moreover,  $\varepsilon_{c, q_h} = -\frac{1}{\sigma}\varepsilon_{U'(c), q_h}$ .<sup>26</sup>

The free entry condition now allows us to link tomorrow's value of money  $\widetilde{\mathbb{E}\phi'}$  to today's movements in  $q_h$ . Free entry gives  $U'(c)q_h = \beta\widetilde{\mathbb{E}\phi'}$  (which is consistent with  $q_h = \beta\widetilde{\mathbb{E}\phi'}$  in the old definition), which means that

$$\varepsilon_{q_h, \beta\widetilde{\mathbb{E}\phi'}} = \frac{1}{\varepsilon_{U'(c), q_h} + 1}, \quad (34)$$

leading to

$$\varepsilon_{c, \beta\widetilde{\mathbb{E}\phi'}} = -\frac{\varepsilon_{U'(c), q_h}}{\sigma(\varepsilon_{U'(c), q_h} + 1)}. \quad (35)$$

To calculate  $\varepsilon_{\phi, \beta\widetilde{\mathbb{E}\phi'}}$ , use

$$U'(c)\phi = p\vartheta u'(q_h) \frac{\beta\widetilde{\mathbb{E}\phi'}}{U'(c)} + (1-p)\beta\widetilde{\mathbb{E}\phi'} = p\vartheta u'(q_h)q_h + (1-p)\beta\widetilde{\mathbb{E}\phi'}, \quad (36)$$

---

<sup>26</sup>Equation (33) captures the general equilibrium effect from changes in  $q_h$ : a shift away from cash consumption will lead to an increase in credit market consumption. This effect is proportional to the share of cash consumption under the binding shock in total consumption. In case of idiosyncratic uncertainty,  $s_{q_h}$  is smaller (by a factor smaller than  $p$ ) than total cash market consumption, and hence the elasticity in equation (33) is smaller.

derived from the FOCs of  $m, q_h$ , to find

$$\begin{aligned} \frac{d \ln \phi}{d \ln \beta \mathbb{E} \phi'} &= \frac{p \vartheta_h u'(q_h) q_h}{p \vartheta_h u'(q_h) q_h + (1-p) \beta \mathbb{E} \phi'} \frac{d \ln(u'(q_h) q_h)}{d \ln q_h} \frac{d \ln q_h}{d \beta \mathbb{E} \phi'} \\ &\quad + \frac{(1-p) \beta \mathbb{E} \phi'}{p \vartheta_h u'(q_h) q_h + (1-p) \beta \mathbb{E} \phi'} - \frac{d \ln U'(c)}{d \ln q_h} \frac{d \ln q_h}{d \ln \mathbb{E} \phi'}, \end{aligned} \quad (37)$$

Combining (37) and (36), we find

$$\varepsilon_{\phi, \mathbb{E} \phi'} = \left( \frac{p+i}{1+i} (1-\sigma) - \varepsilon_{U'(c), q_h} \right) \frac{1}{\varepsilon_{U'(c), q_h} + 1} + \frac{1-p}{1+i}. \quad (38)$$

Likewise, we can calculate

$$\varepsilon_{U'(c) \phi, \mathbb{E} \phi'} = \frac{d \ln U'(c) \phi}{d \ln \beta U'(c') \phi'} = \left( \frac{p+i}{1+i} (1-\sigma) \right) \frac{1}{\varepsilon_{U'(c), q_h} + 1} + \frac{1-p}{1+i}. \quad (39)$$

With  $\varepsilon_{\beta \mathbb{E} \phi', 1+i} = (\varepsilon_{U'(c) \phi, \beta \mathbb{E} \phi'} - 1)^{-1}$  and  $\varepsilon_{U'(c) \phi, \beta \mathbb{E} \phi'}$  from (39), we find

$$\varepsilon_{1+i, \mathbb{E} \phi'} = - \left( \frac{1+i}{p+i} \cdot \frac{\varepsilon_{U'(c), q_h} + 1}{\varepsilon_{U'(c), q_h} + \sigma} \right). \quad (40)$$

Now we are able to calculate the elasticity of velocity with respect to  $\mathbb{E} \phi'$ .

$$\begin{aligned} \varepsilon_{V_c, \beta \mathbb{E} \phi'} &= \frac{d \ln V_c}{d \ln \mathbb{E} \phi'} = - \frac{d \ln V}{d \ln 1 + \varpi} = -\varepsilon_{V_c, 1+\varpi} = s_c \left( \frac{d \ln c}{d \ln \mathbb{E} \phi'} - \frac{d \ln \phi}{d \ln \mathbb{E} \phi'} \right) + s_{cash, nb} \frac{d \ln \psi}{d \ln \mathbb{E} \phi'} \\ &= s_c \left( \left( -\frac{1}{\sigma} + 1 \right) \varepsilon_{U'(c), q_h} - \frac{p+i}{1+i} (1-\sigma) \right) \frac{1}{\varepsilon_{U'(c), q_h} + 1} - \frac{1-p}{1+i} - s_{cash, nb} \frac{1}{\varepsilon_{u'(c), q_h} + 1}. \end{aligned} \quad (41)$$

From (41) it follows that, for  $p = 1$ , this elasticity is negative if  $\sigma < 1$ ; for  $p < 1$ , a larger  $\sigma$  will also lead to a negative elasticity. Combining (40) and (41) yields

$$\begin{aligned} \varepsilon_{V_c, 1+i} &= \left( \frac{1+i}{p+i} \cdot \frac{\varepsilon_{U'(c), q_h} + 1}{\varepsilon_{U'(c), q_h} + \sigma} \right) \times \\ &\quad \left( s_c \left\{ \left[ \left( \frac{1}{\sigma} - 1 \right) \varepsilon_{U'(c), q_h} + \frac{p+i}{1+i} (1-\sigma) \right] \frac{1}{\varepsilon_{U'(c), q_h} + 1} + \frac{1-p}{1+i} \right\} + s_{cash, nb} \frac{1}{\varepsilon_{u'(c), q_h} + 1} \right), \end{aligned} \quad (42)$$

which simplifies to

$$\varepsilon_{V_c, 1+i} = s_c \left( \frac{1}{\sigma} \frac{1+i}{p+i} - 1 \right) + s_{cash, nb} \left( \frac{1+i}{p+i} \right) \left( \frac{1}{\varepsilon_{U'(c), q_h} + \sigma} \right) \quad (43)$$

□

## C A Version of the Model with Interest-Paying Money

We modify our benchmark model by introducing interest payment on money balances carried over from period to period. Thus, we are introducing interest payment on liquid savings. (We have also tried a version where all money is interest-bearing, even within the period; the quantitative implications of that model are similar, but this version is a more natural counterpart of savings accounts in the data, since short-term liquid balances, carried in cash or checking accounts, are not typically interest-bearing.)

Denote by  $i^m$  the interest rate paid on money; the nominal interest rate on bonds becomes  $i^b$ . The aggregate state variables become  $S = (z, i_{-1}^m, i_{-1}^b, i^m, i^b, (1 + \varpi_{-1})\phi_{-1})$ . Firm problem remains the same. Given this, we have the household problem as follows, where only the budget constraint and the exogenous shock processes are affected:

$$V(k, m, b; S) = \max_{c, h, \tilde{m}, k', b', \{q_\vartheta\}} \left\{ U(c) - Ah \right. \quad (44)$$

$$\left. + \mathbb{E}_\vartheta \left[ \vartheta u(q_\vartheta) + \beta \mathbb{E}_{z', i'} V(k', m', \frac{b'}{1 + \varpi}; S') \right] \right\}$$

$$\text{s.t. } c + \phi \tilde{m} + k' + \phi b' = \phi m(1 + i_{-1}^m) + \phi b(1 + i_{-1}^b) + (1 + r - \delta)k + wh \quad (45)$$

$$\psi q_\vartheta \leq \tilde{m} \quad (46)$$

$$\psi = \frac{1 + i^b}{\phi} \quad (47)$$

$$\pi = \frac{(1 + \varpi_{-1})\phi_{-1}}{\phi} \quad (48)$$

$$1 + \varpi = \Omega(S) \quad (49)$$

$$m' = \frac{\tilde{m}}{1 + \varpi} - \frac{\psi q_\vartheta}{1 + \varpi} + \frac{\varpi}{1 + \varpi} \quad (50)$$

$$z' = \xi_{zz}z + \varepsilon'_1 \quad (51)$$

$$\widetilde{(1 + i^{b'})} = \xi_{ii}\widetilde{(1 + i^b)} + \xi_{i\pi}\tilde{\pi} + \xi_{iy}\tilde{y} + \varepsilon'_2 \quad (52)$$

$$\left( \frac{\widetilde{1 + i^m}}{\widetilde{1 + i^b}} \right)' = \xi_{rr} \left( \frac{\widetilde{1 + i^m}}{\widetilde{1 + i^b}} \right) + \xi_{ri}\widetilde{(1 + i^b)} + \xi_{r\pi}\tilde{\pi} + \xi_{ry}\tilde{y} + \varepsilon'_3 \quad (53)$$

The equilibrium conditions for this version of the model are:

$$\begin{aligned}
U'(c) &= \beta \mathbb{E}_{z', i'} [U'(c')(1 + r' - \delta)] \\
U'(c) &= \frac{A}{w} \\
\psi &= \frac{1 + i^b}{\phi} \\
\frac{\mu_{\vartheta}}{U'(c)} &= \mathbb{P}(\vartheta) \left( \frac{\vartheta u'(q_{\vartheta})}{U'(c)\psi} - \frac{\phi(1 + i^m)}{1 + i^b} \right); \tilde{\mu}_{\vartheta}(\tilde{m} - \psi q_{\vartheta}) = 0 \quad \forall \vartheta \\
\phi &= \frac{\sum_{\vartheta} \mu_{\vartheta}}{U'(c)} + \frac{\phi(1 + i^m)}{1 + i^b} \\
\frac{\phi}{1 + i^b} &= \beta \widetilde{\mathbb{E}\phi'} \\
y + (1 - \delta)k &= c + k' + \sum_{\vartheta} \mathbb{P}(\vartheta) q_{\vartheta}
\end{aligned} \tag{54}$$

In terms of dynamics of the model, notice that now, the decisions of the household are often determined not by the dynamics of the nominal bond interest rate, but of the ratio of the bond-to-money interest rates. This is the channel which results in dampened volatilities in this version of the mode (with separable utility). As we show in the text, the CES version of the model, even with interest-paying money, brings the dynamics back to data levels.

## D Computation Procedure

To compute the model we employ the Parameterized Expectations Approach. The method approximates the expectations terms in our Euler equation system (29) - two in total - by polynomial functions of the state variables, and the coefficients of the approximation and solved for. We choose the following forms:

$$\begin{aligned}
\mathbb{E} \left[ (c')^{-\sigma} (1 + e^{z'} \theta(k')^{\theta-1} (h')^{1-\theta} - \delta) \right] &= \psi^1(\chi; \gamma^1) \\
\mathbb{E} \left[ \frac{1}{w'} \right] &= \frac{\widetilde{\mathbb{E}}}{w} = \psi^2(\chi; \gamma^2)
\end{aligned} \tag{55}$$

where

$$\psi^j(\chi; \gamma^j) = \gamma_1^j \exp(\gamma_2^j \log k + \gamma_3^j \log z + \gamma_4^j \log i_{-1} + \gamma_5^j \log i + \gamma_6^j \log[\phi_{-1}(1 + \varpi_{-1})]).$$

The accuracy of approximation can be increased by raising the degree of approximating polynomials above; we have experimented with several forms and found the results robust to them. We find that the convergence properties of our model are good: convergence is monotone and very robust. In order to compute moments from the model, we re-run the model solution 100 times given parameters of the model, and the simulations within each run are for 10,000 periods, where we discard the first 1,000 in computing the moments.

## E Cross-Correlations of Aggregate Variables with Output in Benchmark Model

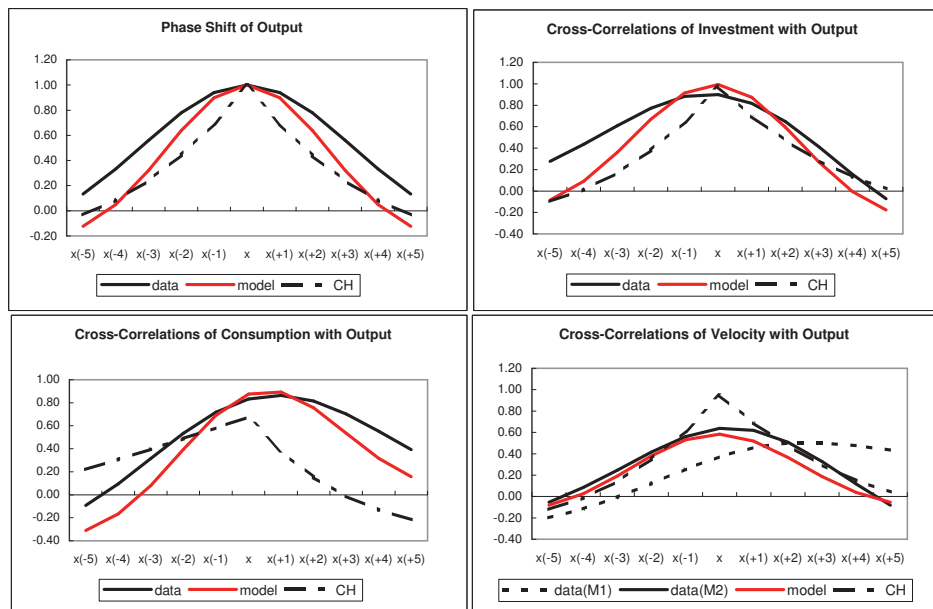


Figure 4: Cross-Correlations of Endogenous Variables with Output

To analyze our performance further with respect to facts highlighted by Cooley and Hansen, we present cross-correlations of several endogenous variables with output in graphical form, in figures 4 - 6. Where possible, we also graph the cross-correlations presented by Cooley and Hansen from their model.<sup>27</sup> With respect to the correlations of real variables with output, we do as well as the Cooley-Hansen model, or better. A notable improvement in our model relative to Cooley and Hansen concerns the dynamic pattern of output velocity (bottom right panel): we match the data for M2 velocity a lot more closely than they did. This last fact is the product of adding precautionary demand for money into the model; we further demonstrate this in figure 5 which shows the same cross-correlations, but comparing our benchmark to our no-shock model. In the bottom right panel, it is clear that the model with preference shocks does a lot better at matching the data than the model without. The other three panels of that figure also show that the improvement in dynamic patterns of real variables relative to Cooley-Hansen results in large part from our driving processes, rather than from preference shocks: we use an interest rate rule, while Cooley and Hansen used a money growth rule.

Finally, in figure 6 we present some further cross-correlations that we get less well. While we get the dynamic pattern of money supply half-right (although our cross-correlation bottoms out later than the data suggest), and we improve on Cooley and Hansen's cross-correlation of nominal interest rates, we get neither these two, nor the dynamic patterns of prices and inflation. Our performance on the bottom two panels is fairly close to Cooley and Hansen's. Again, we do not

<sup>27</sup> Obviously, this comparison is limited in that their model is calibrated to a different time period; we do not present their data for space reasons.



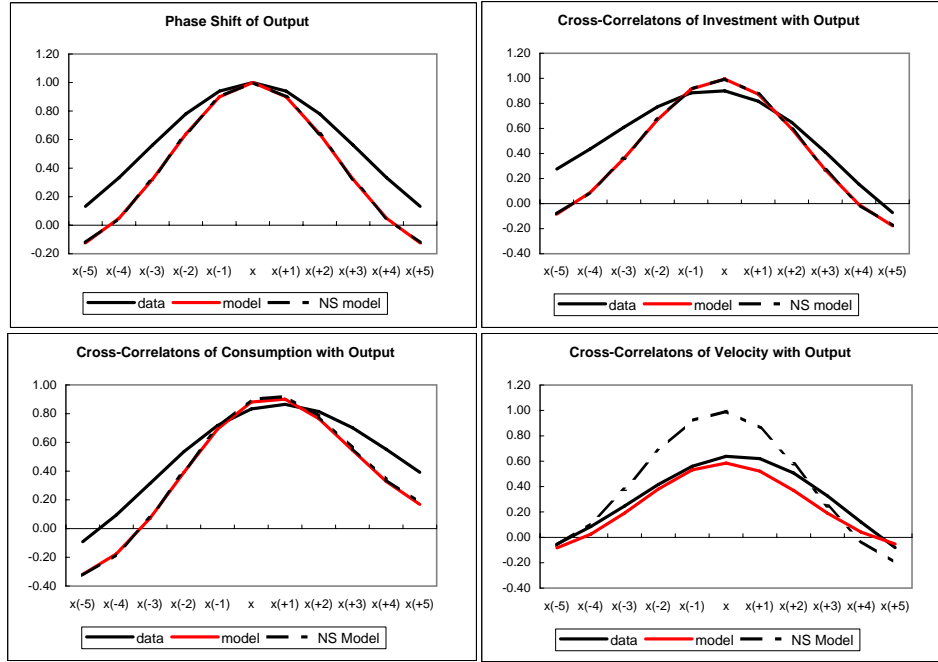


Figure 5: Cross-Correlations of Endogenous Variables with Output, Benchmark vs No-Shock Model

expect to get the patterns of prices to replicate the data with a price adjustment mechanism that is as flexible and frictionless as ours.

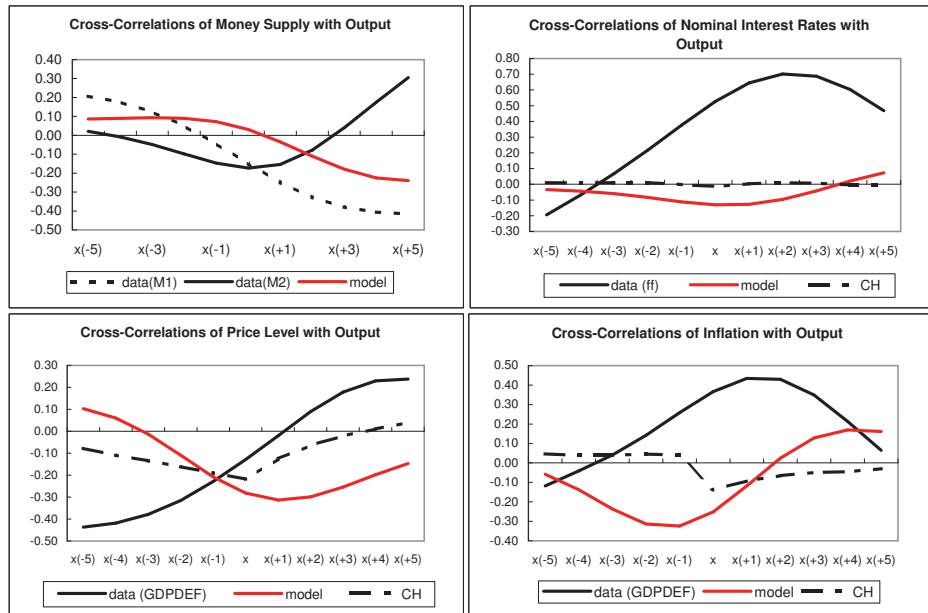


Figure 6: Cross-Correlations of Endogenous Variables with Output

## F Steady State Consumption in Closed Form

We have to solve both for credit market and cash market consumption in order to conduct the welfare cost experiment. From the characterizing equation system (29), we get steady-state credit market consumption, after substituting in the capital-labor ratio, from

$$\bar{c} = \left[ \frac{A}{1-\theta} \left( \frac{1}{\theta} \left( \frac{1}{\beta} - 1 + \delta \right) \right)^{\frac{\theta}{1-\theta}} \right]^{-\frac{1}{\sigma}}.$$

To get cash market consumption we again appeal to the system (29). The issue for the welfare-cost analysis is that as inflation rate increases, more of the discrete shocks cause the cash constraint to bind. Thus, we solve in closed form here for the general case: suppose that the total number of discrete shock states is  $n$  and  $k$  of these shocks, from  $\vartheta_{n-k+1}$  to  $\vartheta_n$ , bind. For any binding shock, the following system holds, given our functional forms:

$$\begin{aligned} \bar{\mu}_{\vartheta_i} &= \mathbb{P}(\vartheta_i) \left( \vartheta_i x_1 \bar{q}_{\vartheta_i}^{1-\sigma} \bar{c}^\sigma - \frac{\bar{\phi}}{1+\bar{i}} \right) \quad \forall i \in \{n-k+1, k\} \\ \bar{\phi} &= \left( \frac{1+\bar{i}}{\bar{i}} \right) \sum_{i=n-k+1}^k \bar{\mu}_{\vartheta_i} \\ \bar{q}_{\vartheta_i} &= \frac{\bar{\phi}}{1+\bar{i}} \quad \forall i \in \{n-k+1, k\}. \end{aligned}$$

From this, one can solve for the relevant  $\mu_{\vartheta_i}$ , which then determine  $\phi$ , and finally  $q$  in all the binding states, which is a function of the nominal interest rate but not of the binding shock level, as expected. Instead, in the remaining (non-binding) states, consumption is given simply by

$$\bar{q}_{\vartheta_i} = (\vartheta_i x_i)^{\frac{1}{\sigma}} \bar{c} \quad \forall i \in \{1, n-k\},$$

and is not a function of the nominal interest rate, but does change with the level of the non-binding shock.

## G A Version of the Model with Inflation Surprises

In this version of the model, we want to change the timing of agents' information. In the benchmark model, there is no *within-period* aggregate uncertainty: agents find out the aggregate state  $S$  at the beginning of the period, and the only uncertainty they face within the period is the idiosyncratic preference uncertainty. In this setup, inflation impacts agents via its level, but its variability does not impact firms or households. Here, we change the model by introducing within-period inflation surprises: now, we will assume that at the beginning of the cash market, households find out not only their preference shocks, but all agents in addition find out the values of the next period's monetary and productivity shocks.

We rewrite the problem, making explicit the dependence on aggregate state where necessary to

make the changed timing clear:

$$V(k, m, b, S) = \max_{c, h, \tilde{m}, k', b', \{q_\vartheta\}} \left\{ U(c) - Ah \right. \quad (56)$$

$$\left. + \mathbb{E}_{\vartheta, S'} \left[ \vartheta u(q_\vartheta(S')) + \beta \mathbb{E}_{S'} V(k', m', \frac{b'}{1+\varpi}, S') \right] \right\}$$

$$\text{s.t. } c(S) + \phi(S)\tilde{m}(S) + k'(S) + \phi b'(S) = \phi m + \phi b(1+i_{-1}) + (1+r-\delta)k + wh(S) \quad (57)$$

$$\mathbb{E}(\psi(S')) = \frac{1+i}{\phi} \quad (58)$$

$$\psi(S')q_\vartheta(S') \leq \tilde{m}(S) \quad (59)$$

$$\pi = \frac{(1+\varpi_{-1})\phi_{-1}}{\phi} \quad (60)$$

$$1+\varpi = \Omega(S) \quad (61)$$

$$m' = \frac{\tilde{m}}{1+\varpi} - \frac{\psi q_\vartheta}{1+\varpi} + \frac{\varpi}{1+\varpi} \quad (62)$$

$$z' = \xi_{zz}z + \varepsilon'_1 \quad (63)$$

$$\widetilde{(1+i')} = \xi_{ii}\widetilde{(1+i)} + \xi_{i\pi}\tilde{\pi} + \xi_{iy}\tilde{y} + \varepsilon'_2 \quad (64)$$

The equilibrium conditions for this model are:

$$U'(c) = \beta \mathbb{E}[U'(c')(1+r'-\delta)] \quad (65)$$

$$U'(c) = \frac{A}{w}$$

$$\mathbb{E}[\psi(S')] = \frac{1+i}{\phi}$$

$$\psi(S')\frac{\mu_\vartheta(S')}{U'(c)} = \mathbb{P}(\vartheta_i, S'|S) \left( \frac{\vartheta u'(q_\vartheta(S'))}{U'(c)} - \beta \mathbb{E} \left[ \frac{\phi(S')\psi(S')}{1+\varpi} \frac{U'(c')}{U'(c)} \right] \right);$$

$$\frac{\mu_\vartheta(S')}{U'(c)}(\tilde{m} - \psi(S')q_\vartheta(S')) = 0, \quad \forall \vartheta, S'$$

$$\phi = \sum_{\vartheta, S'} \frac{\mu_\vartheta(S')}{U'(c)} + \frac{\phi}{1+i}$$

$$\frac{\phi}{1+i} = \beta \mathbb{E}\widetilde{\phi'}$$

$$q(S') = \sum_{\vartheta} \mathbb{P}(\vartheta)q_\vartheta(S') \quad \forall S'$$

$$y + (1-\delta)k = c + k' + q(S')$$

$$z' = \xi_{zz}z + \varepsilon'_1$$

$$\widetilde{(1+i')} = \xi_{ii}\widetilde{(1+i)} + \xi_{i\pi}\tilde{\pi} + \xi_{iy}\tilde{y} + \varepsilon'_2$$

The key change here is that now, firms decide on the amount of good  $q$  to take into the retail market, and households on their money holdings, before they know next period's state, but consumption in the cash market occurs after the next period's aggregate state is revealed. Suppose that agents find out that next period's inflation will be higher. This means that, holding price of the cash good

constant, while constrained households today cannot adjust their consumption, the unconstrained households will want to consume strictly more. But supply of the cash good is fixed, because retailers cannot produce additional goods in the cash market. Thus, in order to clear the market, the cash-good price  $\psi$  will rise in response to the inflation surprise.

What does this imply for agents' welfare? With a surprise increase in inflation, the constrained agents will now have even higher marginal utility, since the price of the cash good is higher, while their money holdings are fixed. The constrained agents will have lower marginal utility. Thus, the distortion in relative marginal utilities of constrained versus unconstrained agents in this version of the model will be even higher.

From this, we conclude that an expected change in inflation will be less detrimental for welfare than an unexpected one, because in the former case, the supply of the cash good also adjusts, while in the latter case, this cannot happen.

In terms of the dynamics of nominal and real moments, the performance of this model is quantitatively similar to the benchmark. In this model, velocity of money becomes more volatile (standard deviation 2.3% versus benchmark's 1.4%), because the unconstrained agents' money demand responds not only to the idiosyncratic shock, but also to the aggregate shock that is realized mid-period. Other than this, however, other moments remain very close to the benchmark dynamics.